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COOPERATIVE EFFECTS AND INTRINSIC OPTICAL BISTABILITY IN COLLECTIONS OF ATOMS

FINAL TECHNICAL REPORT

by

Prof. Yacob Ben-Aryeh

November 1989

United States Army
EUROPEAN RESEARCH OFFICE OF THE U.S. ARMY
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ABSTRACT

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Keywords:

Intrinsic Optical Bistability; Cooperative Effects; Resonance Fluorescence; Quantum Noise.

1. INTRODUCTION

Intrinsic optical bistability (IOB) that is not caused by external feedback such as mirrors, has been the subject of recent intense interest.[1] It was first pointed out by Bowden and Sung[2a] and by Bowden[2b] that optical bistability (OB) may occur for a system comprising a collection of atoms which interact with the electromagnetic field and are driven by an externally applied coherent field without external feedback. The first detailed experimental study of intrinsic optical bistability was conducted almost simultaneously by Hajto and Janossy[3] using amorphous $GeSe_2$, who interpreted their results as due to temperature-dependent induced optical absorption in the material, and Bohnert, Kalt and Klingshirn[4], who used CdS, and also Rossmann, Hennbberger and Voigt[5] using the same material. The process in the first case depends upon absorption due to temperature variation induced by the incident field, whereas the later cases depend upon saturation of absorption due to the generation of carriers in the material, and the IOB has been interpreted[6] as due to bandgap renormalization. Since the earlier works, there have been many theoretical and experimental investigations of various forms of IOB[1].

In certain recent works[7] the nonlinear oscillator model has been used as a prototype medium exhibiting intrinsic bistability. Various effects related to intrinsic bistability have been explored by following this model. On the other hand the two-level system has been used as a prototype medium for describing quantum mechanical effects in intrinsic optical bistability.[8-12] In

previous works we have presented quantum mechanical treatments of mirrorless (intrinsic) optical bistability (IOB) from collections of spatially distributed two-level atoms interacting via the electromagnetic (e.m.) field and driven by an externally applied coherent field.

The aim of the present research project was to study the effects of quantum fluctuations on IOB since these effects have not been studied in the previous works. The bistability effects have a potential significance for developing "optical computers". The use of intrinsic bistability as a switching mechanism will have the following advantages:

a) As the external feedback such as mirrors is not needed, the switching unit can be very small and therefore much less complicated, more efficient and less expensive than that used in a conventional optical bistability.

b) The switching times in intrinsic optical bistability will be short relative to the switching times obtained in optical bistability with mirrors. However, the insertion of noise including especially the quantum fluctuations might destroy the hysteresis cycle behavior of the steady state response. Therefore, we have found it to be of utmost importance to investigate these statistical fluctuations.

Some of the physical effects in cooperative resonance fluorescence are very similar to those of the intrinsic bistability[13–17] as both phenomena are related to the coherent dipole-dipole interactions. An interesting problem in this field which has been studied in the present project is the resonance fluorescence spectra from two two-level atoms including the dipole-dipole interactions.

2. SCIENTIFIC RESULTS

The detailed scientific results of the present project are given in the three articles added in the Appendices. Here we give a short review of our work.

The Langevin equations for intrinsic bistability had been derived in previous works[8-10]. The solution of the equations was based on complete factorization of the dipole operators and then the Langevin terms drop out for the expectation values. Instead of this approximation we use in the present work partial factorization, by which the expectation values are made and factorized in the first stage only relative to the atoms. *The deviation from complete factorization is due to the Langevin noise terms.*

After using the above approximation the quantum nature of the equations is related to the "Langevin force" operators $f^{(+)}(\tau)$, which stem from the vacuum contribution to the fluctuations. τ is the retarded time and $< [f^{(+)}(\tau),\ f^{(-)}(\tau')] > \propto \delta(\tau - \tau')$. The $f(\pm)$ operators are independent of the space coordinate. Therefore the treatment of quantum fluctuations in the case of a short sample (a width smaller than a wavelength) and in the case of a long sample are similar. We assume that the $f(\pm)$ operators are gaussian operators representing a gaussian white noise. By using this approximation we transform the equations for the inversion of population and for the two components of the complex dipole into three coupled Ito random differential equations[18].

The quantum statistical treatment of the intrinsic bistability can be simplified for the case when the dephasing constant is much larger than the

relaxation constant ($\beta_2 >> \beta_1$). In a pure quantum mechanical model $\beta_2 = \beta_1/2$, but we may introduce homogeneous broadening by considering β_1 and β_2 as empirical cases. In many experimental cases the condition $\beta_2 >> \beta_1$ is fulfilled. Under these conditions the relaxation of the dipole operators is very rapid relative to the relaxation of the inversion of population. Therefore for this case the expectation value for the complex dipole operators is adiabatically eliminated by using Eq. (17) of Ref. 9. We get by this approximation a one-dimensional Ito differential equation for the inversion of population. The methods for solving such a random differential equation have been described in the theories developed for bistability in laser cavities[18-20]. The tunnelling process and the passage times between the two stable states in intrinsic bistability were calculated by using these methods for such cases in which $\beta_2 >> \beta_1$. The analytical and numerical calculations made by following this method, are presented in Appendix A. Cooperative effects in the resonance spectra of two-level atoms are usually treated by assuming that the atoms are within a small volume with dimensions smaller than a wavelength[21-22]. In Appendix B we analyze the resonance fluorescence spectra of two two-level atoms by taking into account the dependence of the coherent dipole-dipole interactions on the distance between the atoms.

In the present project we have treated also a pure quantum mechanical system in which the relaxation rate β_1 of the inversion of population is the spontaneous decay time and the relaxation rate for the dipole moment is given by $\beta_2 = \beta_1/2$. By using the quantum theory of noise developed by

Lax[23] we have calculated al the second moments of the dipole operators and studied the statistical properties of the reaction field.

It has been shown by other authors that squeezing[24–26] can be produced by optical bistability[27] but this possibility has been studied only under the approximation of a good cavity limit. For such cases the medium relaxation is much faster than that of the field and then the atomic variables can be eliminated. The problem of photon statistics is studied in Appendix C for the extreme case of a bad cavity limit where we preclude any reflecting boundaries and where the relaxation of the field is much faster than the relaxation of the material. The statistical properties of the transmitted radiation are discussed and it has been found that squeezing is not obtained in the present system.

CONCLUSIONS

The simple modified Maxwell-Bloch equations used by us are useful as a generic model for describing the effects of quantum fluctuations on intrinsic bistability. The long passage times calculated by us, for a two-level system with large dephasing rates indicate reasonably suitable conditions for obtaining intrinsic optical bistability. However, the effect of quantum fluctuations in terms of the average passage times can be an important effect. Quantum noise effects, as discussed here, can lead to significant "glitch" rates which can seriously affect stability and switching properties of a bistable device.

By solving our equations for the pure quantum mechanical case we find that the reaction field in intrinsic bistability is bunched so that a quantum

noise which is above the quantum noise limit is inherent in this system. We conclude that the photon statistics in the bad cavity limit is more chaotic than the good cavity limit in which squeezing was predicted.

The theoretical analysis made in the present work for the resonance fluorescence spectra of driven two, two-level atoms can be used for evaluating the spectra in numerical calculations. For the case of two, two-level atoms at resonance interacting with a resonant external field, which are within a volume with a dimension smaller than a wavelength, we have derived an analytical expression for the line shape. Additional side bands appear, which are shifted from the line center approximately by twice the Rabi frequency, but these side bands are very broad and very weak.

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APPENDIX A

Quantum Noise Effects in Intrinsic Optical Bistability of A System of Interacting Two-Level Atoms.

QUANTUM NOISE EFFECTS IN INTRINSIC OPTICAL BISTABILITY OF A SYSTEM OF INTERACTING TWO-LEVEL ATOMS

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Langevin equations for intrinsic optical bistability (IOB) in a system of two-level atoms interacting by dipole–dipole interaction are treated, and the stochastic effects of quantum noise are analyzed. We derive a one-dimensional Ito stochastic differential equation for the inversion of population of cases in which the population relaxation is very slow relative to the homogeneous dephasing rate of the complex dipole. For such cases the probability distribution, diffusion process and the first average passage times between the two metastable states in IOB are calculated.

Various forms of intrinsic optical bistability (IOB), i.e. optical bistability that does not depend upon external feedback such as mirrors, have been investigated in many theoretical and experimental works [1–11]. Thermally induced IOB has been obtained experimentally and has been related to internal feedback due to nonlinear absorption increasing as a function of intensity [2,8,9]. Dagenais and Sharfin [10] observed IOB by tuning just below the absorption peak of the bound I_2 exciton in CdS. This was due to thermally induced shift of the exciton resonance to low frequency (a frequency renormalization which is intensity dependent).

Non-thermal IOB effects were observed by Bohnert et al. [3] and by Rossmann et al. [4]. These experiments were explained by Schmidt et al. [5] as due to Coulomb screening of carriers caused by laser field induced carrier production resulting in shrinkage of the band gap. In this work the tuning is near the band edge in CdS and is entirely an effect of many body interaction in the electron-hole plasma.

Currently IOB is routinely observed or reported in the literature. What has not been reported and appears relatively difficult is the observation of IOB in a system which can be quantitatively analyzed using a simple analytical model. In this context our simple modified Maxwell Bloch equations [12–14] are useful as a generic model. One should distinguish between IOB which is treated in a steady state and superfluorescence (SF) [15] which is a transient phenomenon depending on spontaneous initiation and subsequent coherent pulse buildup. Unlike SF, IOB has an externally applied field which, when linearly polarized, causes individual dipole orientation in the medium and the direction of the reaction field of the atoms is largely determined by the incident driving field, into the appropriate narrow solid angle. Therefore, the dynamics of switching in IOB is quite different from that of SF in terms of initial and boundary conditions. In a small volume ($<\lambda^3$) SF cannot happen since there is no preferred direction. Whether or not IOB can happen for interacting two-

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level atoms separated by less than a resonance wavelength, has been strongly addressed in the calculational heuristic treatment of Hopf and Bowden [7]. In that work it has been shown that the stochastic processes in the two-level atoms with the dipole–dipole interactions causes bimodality. A "clean experiment" which might be quite close to this model, would be a collection of Rydberg atoms [16] in ion traps [17,18]. The collisional broadening can be controlled by laser cooling and decorrelation can be controlled by introduction of thermal IR or FIR radiation. Thus, atoms can be brought closer together by laser cooling and decorrelated independently by interacting with an externally applied stochastic field superimposed on the coherent driving field (this needs further considerations). As has been shown in the heuristic treatment of ref. [7] bistability is obtained when the number of atoms within a cubic wavelength is large.

In recent works [12–14,19] we have treated the problem of IOB in collections of spatially distributed two-level atoms interacting via the electromagnetic field and driven by an externally applied coherent field. In the earlier works on IOB of two-level systems the dipole–dipole interactions have been treated by assuming a small volume with dimensions smaller than a resonance wavelength [1,6] or by assuming a system with a small number of atoms in semiclassical approximation [7]. In the recent works [12–14,19] we have treated the problem from the many-body standpoint by developing the Heisenberg equations of motion in the "bad-cavity" limit, where the relaxation of the field in the medium is much faster than the relaxation of the material. This is the case when the cavity width γ is very large. Since γ^{-1} corresponds to the average photon lifetime in the cavity we obtain the extreme "bad cavity limit" by precluding any reflecting boundaries and when the upper bound for the photon lifetime is given by the passage time through the medium. If the cavity width γ is much larger than the material relaxation rate then the field variables can be adiabatically eliminated from the Heisenberg equations of motion with the approximation that the field is in steady state on the time scale of the material response time. The formal procedure for adiabatic elimination, specifically in the "bad cavity limit" as well as the "good cavity limit", has been presented by Lugiato [20]. The "good cavity limit" is of course the complement of the "bad cavity" approximation and requires that the reflectivity boundary condition prevails, i.e. a high-Q cavity. In this case the photon lifetime is large and the cavity width is small. Thus it can happen that the medium relaxation is much faster than that of the field and then the atomic variables can be adiabatically eliminated [20,21]. An example would be a nonlinear cavity composed of a Kerr medium between highly reflecting mirrors. In our works we have treated short samples configurations of two-level atoms without any reflecting boundary conditions so that we have assumed the "bad cavity limit".

In our previous works [12–14] we have shown that cubic nonlinearities appearing in dense two-level systems, due to the dipole–dipole interactions, can lead to IOB. This, of course, is related to early quantum treatments of bistability that is in terms of factorization and cubic nonlinearities [22,23]. The aim of the present Letter is to describe the effect of quantum noise on IOB. As is well known the stochastic processes, in general, change the phenomenon of bistability to bimodal probability distributions. Since stochastic effects have not been treated previously for IOB our new analysis of such effects should be of interest.

The equations of motion obtained for a short sample of two level atoms, including the noise source operators, are given as [12–14]

$$\begin{aligned} \mathrm{d}\langle\sigma_z(z,\tau)\rangle_a/\mathrm{d}\tau &= -\beta_1[\langle\sigma_z,\tau)\rangle_a + n] \\ &+ (\mu/\hbar)\,[E^*\langle\sigma_{+0}(z,\tau)\rangle_a + E\langle\sigma_{-0}(z,\tau)\rangle_a] \\ &- 2\langle\sigma_{+0}(z,\tau)\rangle_a f^+(\tau) - 2f^-(\tau)\,\langle\sigma_{-0}(z,\tau)\rangle_a , \end{aligned} \tag{1}$$

$$\begin{aligned} \mathrm{d}\langle\sigma_{+0}(z,\tau)\rangle_a/\mathrm{d}\tau &= -(\mu E/2\hbar)\,\langle\sigma_z(z,\tau)\rangle_a \\ &+ \mathrm{i}[\Delta - \epsilon\langle\sigma_z(z,\tau)\rangle_a]\langle\sigma_{+0}(z,\tau)\rangle_a \\ &- \beta_2\langle\sigma_{+0}(z,\tau)\rangle_a + f^-(\tau)\,\langle\sigma_z(z,\tau)\rangle_a . \end{aligned} \tag{2}$$

Here $\langle\sigma_{+0}\rangle_a$ is the expectation value of the slowly varying complex atomic polarization per unit volume, and $\langle\sigma_z\rangle_a$ is the expectation value of the atomic inversion per unit volume. The average $\langle\ \rangle_a$ are with respect to the atomic states only. The parameter μ is the modulus of the matrix element of the transition dipole moment of an atom, and n is the number of atoms per unit volume. $\Delta = \omega - \omega_0$ is the detuning

where ω is the frequency of the applied field E and ω_0 is the atomic resonance frequency. The rates β_1 and β_2 are the inverses of the relaxation times T_1 and T_2 for the inversion of population and the dipole moment, respectively, and $\Delta=\omega-\omega_0$ is the deviation of the applied field frequency ω from the atomic resonance frequency ω_0. ϵ is the frequency renormalization constant derived in our previous work

$$\epsilon=7\pi n\beta c^3/4\omega_0^3, \tag{3}$$

where β is the spontaneous decay constant

$$\beta=4\mu^2\omega_0^3/3\hbar c^3. \tag{4}$$

The frequency renormalization stems from coherent dipole–dipole interactions between atoms that are within a volume of a cubic wavelength. The electromagnetic field appears in eqs. (1), (2) in two parts, i.e. the free field f and the macroscopic field E. The latter is assumed to be a constant classical driving field (or equivalently that it is in coherent state). The free field operators of eqs. (1), (2) obey in the plane wave limit, the equations

$$\begin{aligned}&\langle f^+(\tau)f^-(\tau')\rangle=\beta\delta(\tau-\tau'),\\&\langle f^+(\tau)f^+(\tau')\rangle=\langle f^-(\tau)f^-(\tau')\rangle\\&\quad=\langle f^-(\tau)f^+(\tau')\rangle=0,\end{aligned} \tag{5}$$

where the averages $\langle\ \rangle$ are expectation values, and τ is the retarded time variable $\tau=t-z/c$. These operators constitute fluctuating noise terms which stem from the quantum field coupling to the atoms.

In our previous works we have treated the problem of factorization of products of atomic operators that arise from adiabatic elimination of the reaction field contributions. Here the multiplications with the slowly varying external field E are automatically factorized under the present assumption of a driving classical field. The only operator product terms left in eq. (1) are the multiplicative noise terms, and this of course factorize with respect to the atomic initial states (in consistence with the markovian approximation of eq. (5)). In the present work we assume also adiabatic elimination conditions for $\langle\sigma_{+0}\rangle_a$ which means overwhelming large dephasing of $\langle\sigma_{+0}\rangle$ ($\beta_2\gg\beta_1$). Thus, under these conditions, the coherence of the off-diagonal dipole terms is quenched and the diagonal and off-diagonal terms are decorrelated. Thus, under the adiabatic approximation for strong dephasing, decorrelation of σ_{+0} and σ_z is an excellent approximation, and indeed, must follow for consistency. One should be aware of the fact that the effect of IOB treated here is not related at all to the cooperative effects discussed for SF. It is related to the renormalization of the frequency due to the high density of the two level atoms and is similar to the renormalization of frequency for IOB in semiconductors as explained in the introduction to this Letter.

By adiabatic elimination of $\langle\sigma_{+0}\rangle$ and $\langle\sigma_{-0}\rangle$ in eqs. (1), (2) we obtain the single Ito equation [24,25] for the inversion

$$\begin{aligned}\mathrm{d}\sigma_z=&\left(-\beta_1(\sigma_z+1)-\frac{|\omega_R|^2\sigma_z\beta_2}{(\Delta-\epsilon\sigma_z)+\beta_2}\right)\mathrm{d}\tau\\&-\left(\frac{-\omega_R\sigma_z}{\mathrm{i}(\Delta-\epsilon\sigma_z)}\mathrm{d}F^+ + \frac{\omega_R^*\sigma_z}{\mathrm{i}(\Delta-\epsilon\sigma_z)+\beta_2}\mathrm{d}F^-\right),\end{aligned} \tag{6}$$

where $\omega_R=\mu E/\hbar$ is the Rabi frequency of the applied external field. Here, we have defined the dimensionless stochastic variables $\mathrm{d}F^+$ and $\mathrm{d}F^-$ by the equations

$$\frac{\mathrm{d}F^+(\tau)}{\mathrm{d}\tau}=f^+(\tau),\quad \frac{\mathrm{d}F^-(\tau)}{\mathrm{d}\tau}=f^-(\tau), \tag{7}$$

and have dropped the $\langle\ \rangle_a$ notation for convenience and scaled the atomic variables to functions corresponding to densities per atom. The stochastic properties of the variables $\mathrm{d}F^+$ and $\mathrm{d}F^-$ follow from eqs. (5) and (7),

$$\begin{aligned}&\langle\mathrm{d}F^+(\tau)\,\mathrm{d}F^-(\tau')\rangle=\beta\,\mathrm{d}\tau,\\&\langle\mathrm{d}F^+(\tau)\,\mathrm{d}F^+(\tau')\rangle=\langle\mathrm{d}F^-(\tau)\,\mathrm{d}F^-(\tau')\rangle\\&\quad=\langle\mathrm{d}F^-(\tau)\,\mathrm{d}F^+(\tau')\rangle=0.\end{aligned} \tag{8}$$

The model can be easily generalized to other specific radiation reservoir conditions, such as a thermal reservoir at temperature $T\neq0$. Using conditions (8) we transform eq. (6) into a Fokker-Planck equation [24,25] for the inversion of population σ_z as a function of the parameters $|\omega_R|^2$, Δ, ϵ, β_1 and β_2 (where $\beta_2\gg\beta_1$):

$$\frac{\partial f(\sigma_z)}{\partial \tau} = -\frac{\partial}{\partial \sigma_z}\left[\left(-\beta_1(\sigma_z+1) - \frac{|\omega_R|^2\sigma_z\beta_1}{\beta_2^2+(\Delta-\epsilon\sigma_z)^2}\right)f(\sigma_z)\right] + \frac{1}{2}\frac{\partial^2}{\partial\sigma_z^2}\left(\frac{|\omega_R|^2\sigma_z^2\beta_1}{\beta_2^2+(\Delta-\epsilon\sigma_z)^2}f(\sigma_z)\right). \quad (9)$$

We represent the drift term $A(\sigma_z)$ and the diffusion coefficient $B(\sigma_z)$ as

$$A(\sigma_z) = -\beta_1(\sigma_z+1) - \frac{|\omega_R|^2\sigma_z\beta_2}{\beta_2^2+(\Delta-\epsilon\sigma_z)^2}, \quad (10)$$

$$B(\sigma_z) = \frac{|\omega_R|^2\sigma_z^2\beta_1}{\beta_2^2+(\Delta-\epsilon\sigma_z)^2}. \quad (11)$$

The steady state solution of (9) is given by

$$f^{(st)}(\sigma_z) = C\frac{1}{B(\sigma_z)}\exp\left(2\int_{\sigma_z=-1}^{\sigma_z} d\sigma_z\frac{A(\sigma_z)}{B(\sigma_z)}\right) = \frac{C}{B(\sigma_z)}\exp[-U(\sigma_z)], \quad (12)$$

where C is a normalization constant and the potential $U(\sigma_z)$ is defined as

$$U(\sigma_z) = -2\int_{\sigma_z=-1}^{\sigma_z} d\sigma_z\frac{A(\sigma_z)}{B(\sigma_z)}. \quad (13)$$

$U(\sigma_z)$ plays a role similar to a chemical potential since the main dependence on σ_z is through this function while $B(\sigma_z)$ is a slowly varying function of σ_z.

By a straightforward integration we get

$$U(\sigma_z) = \ln(|\sigma_z|)\left(\frac{2\beta_2}{\beta_1}+\frac{2(\beta_2^2+\Delta^2)-4\Delta\epsilon}{|\omega_R|^2}\right) + (1+\sigma_z)\frac{2\epsilon^2-4\Delta\epsilon}{|\omega_R|^2} + (\sigma_z^2-1)\frac{\epsilon^2}{|\omega_R|^2} - \frac{2(\beta_2^2+\Delta^2)}{|\omega_R|^2}\left(\frac{1}{\sigma_z}+1\right). \quad (14)$$

To illustrate the quantum noise effects in intrinsic bistability we show in fig. 1 the value of $\langle\sigma_z\rangle$ as a function of $|\omega_R|^2$, where all rates are normalized to β_1. This figure has been calculated according to the steady state cubic equation of state obtained in our previous work (ref. [12], eq. (18)), and this, of course, is in total agreement with the expression for the drift term of eq. (10). The dephasing rate, β_2, has been chosen to be 10 β_1, so that the approximation $\beta_2 \gg \beta_1$ is justified. To obtain a condition of intrinsic bistability the parameter ϵ must satisfy [6,14] $\epsilon/\beta_2 > 6$. For illustration we have chosen $\epsilon = 100$ and $\Delta = -10$, in units of β_1. This gives the ratio $\epsilon/\beta_2 = 10$. The solid and dotted curves in fig. 1 represent, respectively, stable and unstable steady states. The points a, b and c in this figure represent, respectively, the three steady state solutions for σ_z for a prescribed field value $|\omega_R|^2$, where $j = 1, 2, 3$ refer to $|\omega_R|^2 = 100, 120, 150$ respectively. For the same conditions and prescribed fields the corresponding probability functions, $f^{(st)}(\sigma_z)$, calculated from eq. (12), are depicted as functions of σ_z in fig. 2. The maxima of the probability density functions, in each case, correspond to the two stable solutions, a and c, of the cubic equation whereas the minima, b, correspond to the associated unstable solution of the equation.

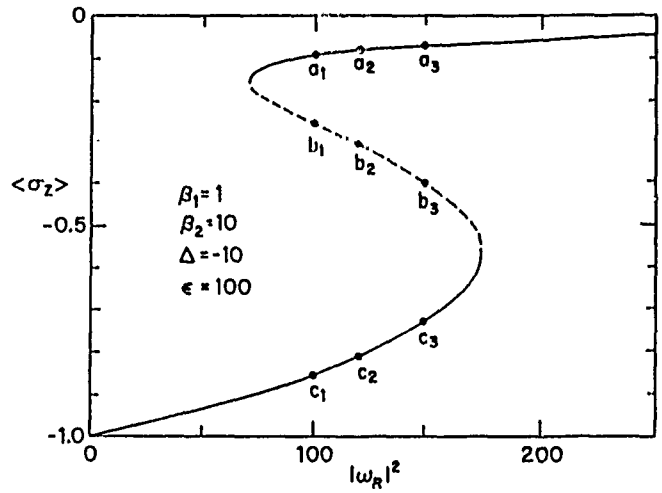

Fig. 1. $\langle\sigma_z\rangle$ as a function of $|\omega_R|^2$ for normalized parameters $\beta_1 = 1$, $\beta_2 = 10$, $\Delta = -10$ and $\epsilon = 100$, where all quantities are in units of β_1. The solid and dotted curves represent respectively stable and unstable steady states. The points a, b and c represent the three solutions of the cubic steady state equation where the subscripts $j = 1$, 2 and 3 refer, respectively, to external fields $|\omega_R|^2 = 100$, 120 and 150 in units of β_1.

The time dependent Fokker-Planck equation, eq. (9), can be described by [20]

$$\partial f(\sigma_z, \tau)/\partial\tau = \lambda f(\sigma_z, \tau), \quad (15)$$

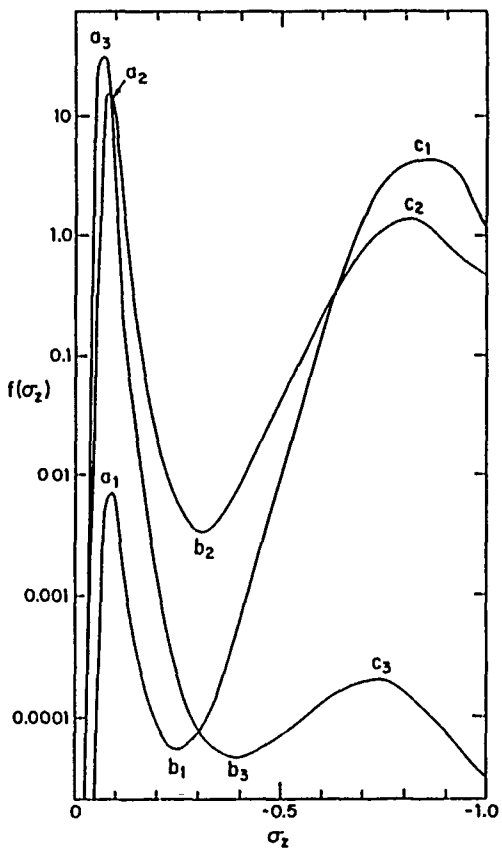

Fig. 2. The probability density $f(\sigma_z)$ for various external fields as a function of σ_z for stationary states with normalized parameters $\beta_1=1$, $\beta_2=10$, $J=-10$, $\epsilon=100$. The three curves are for fields with $|\omega_R|^2=100$, 120 and 150. The maxima a and c correspond to the stable solutions of the cubic equations and the minima b to the corresponding unstable solutions, where $j=1$, 2, 3 refer to $|\omega_R|^2=100$, 120 and 150, respectively. All parameters are in units of β_1.

where, under conditions similar to those which pertain to fig. 2, the linear operator exhibits a nearly degenerate pair of lowest lying eigenvalues, $\lambda_0=0$ and $\lambda_1\neq 0$, and a large gap between λ_1 and the remaining eigenvalues. The time evolution of $f(\sigma_z, \tau)$ occurs in two separate steps. In the first one the relaxation process is local in which the two peaks of the double peaked distribution described in fig. 2, evolve independently of each other [20]. This stage is very rapid corresponding to the high eigenvalues for which $\lambda_j \gg \lambda_1$, $(j>1)$. The shape of the two peaks, of a given distribution at the end of the first stage is identical to that represented by $f^{(st)}(\sigma_z)$, but the ratio of the maxima of the peaks may be different, depending on the initial conditions. Referring to fig. 1, we may start in the upper (highly excited) branch of σ_z and change $|\omega_R|^2$ in the neighborhood of a to approach the point a, or we may start on the lower branch and arrive at the point c. After the fast relaxation of the first step the probability density $f(\sigma_z)$ is given by the line profile about the point a, whereas for the second case it is given by the line profile about point c, as illustrated in fig. 2.

In the long time limit (the second stage) the function $f(\sigma_z)$ can be approximated by

$$f(\sigma_z, \tau)=f^{(st)}(\sigma_z)+\exp(-\lambda_1\tau)f_1(\sigma_z)\,, \qquad (16)$$

where $f_1(\sigma_z)$ is the eigenfunction of the Fokker-Planck operator corresponding to the eigenvalue λ_1. The probability to find the system in the states a and c at time τ are given, respectively, by

$$w_a(\tau)=\int_{\sigma_z(b)}^{0} f(\sigma_z)\,d\sigma_z\,,$$

$$w_c(\tau)=1-w_a(\tau)\,. \qquad (17)$$

The rate by which $w_a(\tau)$ changes is given by

$$dw_a(\tau)/d\tau=-(1/\tau_a)w_a+(1/\tau_c)(1-w_a)\,, \qquad (18)$$

where τ_a and τ_c are the "escape times" out of the a and c wells of the potential. On the other hand we have

$$dw_a/d\tau=-\lambda_1[w_a(\tau)-w_a(\infty)]\,. \qquad (19)$$

By comparing eqs. (18) and (19) we get

$$\lambda_1=\frac{1}{\tau_a}+\frac{1}{\tau_c}\,,\quad \frac{\tau_a}{\tau_c}=\frac{w_a(\infty)}{w_c(\infty)}\,,\quad \frac{1}{\tau_c}=\lambda_1 w_a(\infty)\,. \qquad (20)$$

By using methods described in refs. [20,21] and approximations that are valid within the switching regime, the "escape times" are given by [20]

$$\tau_a=M\int_{\sigma_z(b)}^{0} d\sigma_z\,[B(\sigma_z)]^{-1}\exp[-U(\sigma_z)]\,,$$

$$\tau_c=M\int_{-1}^{\sigma_z(b)} d\sigma_z\,[B(\sigma_z)]^{-1}\exp[-U(\sigma_z)]\,, \qquad (21)$$

where M is a function of the potential barrier between the two potential wells,

$$M=2\int_{c}^{a} d\sigma_z \exp[U(\sigma_z)]\,. \qquad (22)$$

By calculating the escape times according to the above equations for the three sets of conditions illustrated in figs. 1, 2, we obtain $\tau(a_2)\approx 3.0\times 10^3\ \beta_1^{-1}$, $\tau(c_2)\approx 1.65\times 10^3\ \beta_1^{-1}$; $\tau(a_1)\approx 92.6\ \beta_1^{-1}$, $\tau(c_1)\approx 3.08\times 10^5\ \beta_1^{-1}$; $\tau(a_3)\approx 5.6\times 10^5\ \beta_1^{-1}$, $\tau(c_3)\approx 36.7\beta_1^{-1}$. For the probability density corresponding to $|\omega_R|^2=120$, labelled $j=2$ in fig. 2, we obtain a bimodal distribution and long passage times. This condition corresponds very nearly to a generalized Maxwell's construction and the calculated passage times are very close to the maximum average passage time for the parameters chosen. In regard to the other two cases illustrated in figs. 1, 2 for $j=1$, 3 corresponding to $|\omega_R|^2=100$, 150 respectively, the probability densities are nearly monomodal with drastically disproportionate escape times corresponding to the two stable steady state regions.

The long passage times associated with the regime of clearly distinct bimodality indicate reasonably suitable conditions for observation of IOB in principle, but the effect of quantum fluctuations is a measurable quantity in terms of the average first passage times and can be an important effect, especially in the approach to nearly monomodal behavior near the steady state switching thresholds. Quantum noise effects, as discussed here, can lead to significant "glitch" rates which can seriously affect stability and switching properties of a bistable device.

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APPENDIX B

Resonance Fluorescence Spectra of Two Driven Two-Level Atoms

Resonance Fluorescence Spectra of Two Driven Two-Level Atoms

YACOB BEN-ARYEH AND CHARLES M. BOWDEN

Abstract—We develop a theoretical analysis for the spectrum of the light scattered by two, two-level atoms driven near resonance by an external coherent electromagnetic field. The atoms are assumed to relax to equilibrium with the driving field due to radiation damping. The explicit dependence of the coherent dipole-dipole interactions on the distance between the two atoms is taken into account in the theoretical treatment of the spectra. For the case of two, two-level atoms with a fixed distance smaller than a wavelength, an analytical result for the resonance fluorescence line shape is obtained, and the result is compared to previous works.

I. Introduction

ONE of the most interesting subjects in quantum optics which has been studied extensively in the literature is the resonance fluorescence spectra from a two-level atom interacting with a nearly resonant coherent external field [1]–[5]. Interesting quantum effects, such as photon antibunching, have been observed [6] in connection with the resonance fluorescence phenomena. The structure of the spectrum of resonance fluorescence of a cooperative system of atoms has been investigated in many works [7]–[13]. Senitzky [7] was the first to predict that the spectrum of a cooperative system would consist not only of the usual peaks, but also of additional side bands at the harmonics of the Rabi frequency. This problem has been investigated further in the various works [8]–[13], and in particular, the dynamics of the two, two-level atoms has been investigated.

The purpose of the present work is to study resonance fluorescence spectra of two, two-level atoms by the use of new methods and compare the results to other works.

The spectral distribution of the resonance fluorescence scattered light of one two-level atom is well known [1]. It has been well established experimentally [14] that for low incident field strengths, the scattered spectrum consists of a single broadened line, which above certain threshold strength, it exhibits a pair of side bands in addition to the central component. The ratio of the central line to the side band peak heights is 3 : 1, while the line width ratio is 1 : 1.5. This kind of spectrum is in complete agreement with that of Mollow [1], Oliver *et al.* [15], Carmichael and Walls [4], Kimble *et al.* [6], and many others. This result is in contrast with the one-photon approximation [16], which gives a ratio 2 : 1 rather than 3 : 1 in the peak heights. A more sophisticated one-photon model for resonance fluorescence spectra of a two-level atom has been developed by Baklanov [17] and by Stenholm [18]. This new one-photon model gives the correct resonance fluorescence spectra and is in complete agreement with other works [1], [4], [6], [15]. It is the purpose of the present work to apply this approach [17], [18] for developing the theoretical analysis of the resonance fluorescence spectra of two, two-level atoms and to compare the results to those of other authors [8]–[13].

Cooperative atomic effects in the resonance fluorescence spectra are usually treated by assuming that the atoms are within a small volume with dimensions smaller than a wavelength. The nature of the coherent dipole-dipole interactions between two identical two-level atoms at a fixed arbitrary distance r apart has been studied by Milonni and Knight [19]. The coherent dipole-dipole interactions with their explicit spatial dependence may explain various cooperative effects, such as superradiance [20] and intrinsic bistability [21]. In the present work, we analyze the resonance fluorescence spectra by taking into account the dependence of the coherent dipole-dipole interactions on the distance r between the atoms.

In Section II, we present the general theory for the dynamics of driven, spatially separated two, two-level atoms. We follow here the approach given in [19], by which the dipole-dipole interactions lead to a damping mechanism, but in addition, we include here the effect of the coherent interaction with the externally applied field.

In Section III, we develop the density matrix equations of motion by generalizing the calculations made in [17] and [18] for one atom to two, two-level atoms. The analysis for the resonance fluorescence spectra is made in Section IV. Although for the general cases it was not possible for us to obtain final analytical results, the equations developed by us can be used further in a numerical analysis. Also, we have emphasized the analysis of specific cases for which final physical conclusions have been made.

II. General Theory

Our system is composed of two identical isolated two-level atoms fixed in position with a distance r apart, driven by a strong prescribed incident coherent electromagnetic

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field, which is nearly resonant with the two-level atomic frequency. It is assumed that the two atoms come into equilibrium with the driving field through the effect of radiation damping.

The Hamiltonian, which describes the system, is given in the rotating wave and in the electric dipole approximation by $H = H_0 + H' + H''$:

$$H_0 = \frac{\hbar\omega}{2}\sum_{i=1}^{2}\sigma_z^{(i)} + \hbar\sum_q \omega_q a_q^+ a_q$$

$$H' = -i\hbar\sum_{i=1}^{2}\sum_q \xi_q^{(i)} a_q^+ \sigma_-^{(i)} e^{i\vec{k}_q\cdot\vec{r}_i} + \text{HC}$$

$$H'' = \frac{-i\hbar}{2}\sum_{i=1}^{2}\omega_R\sigma_+^{(i)}e^{-i(\omega_0 t - \vec{k}_0\cdot\vec{r}_i)} + \text{HC} \quad (2.1)$$

where HC denotes the Hermitial conjugate and

$$\xi_q^{(i)} = \left(\frac{2\pi\omega_q}{\hbar V}\right)\vec{\mu}_i\cdot\hat{e}_q. \quad (2.2)$$

H_0 includes the free atoms and free field Hamiltonian. We use here the usual operators related to the SU(2) algebra for a single atom: $\sigma_z^{(i)}$ represents the population inversion operator and $\sigma_\pm^{(i)}$ are the raising and lowering operators for atom i with coordinates $\vec{r}_i$. ω_R is the Rabi rate associated with the applied coherent field. ω_0 and $\vec{k}_0$ are the carrier frequency and wave vector of the applied field, respectively. V is the quantization volume for the field, ω_q is the frequency and $\hat{e}_q$ is a unit vector in the direction of polarization for the mode q, $\vec{\mu}_i$ is the dipole moment vector of atom i, and ω is the atomic frequency of the two-level system.

We limit the discussion in the present paper to the case of an external field propagating perpendicular to the interatomic axis. By choosing the center of coordinates at the center of the interatomic axis, one finds immediately that the spatial factors of H'' vanish. Therefore, for this case, the interaction with the radiation is completely symmetric relative to the two atoms and the analysis of the resonance fluorescence spectra is relatively simpler. The treatment of the general case where the radiation is propagating in any direction can be done in a straightforward way by following the present methods, although the expressions will be more complicated (the interaction H'' inserts spatial factors in the matrix elements which are different for the two atoms). For cases for which $\vec{k}_0\cdot\vec{r} \ll 1$, the spatial factors of H'' can be neglected and the present analysis remains valid for any direction of the external applied field.

A complete set of the atomic states can be given as

$$|\phi_1\rangle = |+\rangle_1|+\rangle_2;$$

$$|\phi_2\rangle = \frac{1}{\sqrt{2}}[|+\rangle_1|-\rangle_2 + |-\rangle_1|+\rangle_2];$$

$$|\phi_3\rangle = \frac{1}{\sqrt{2}}[|+\rangle_1|-\rangle_2 - |-\rangle_1|+\rangle_2];$$

$$|\phi_4\rangle = |-\rangle_1|-\rangle_2 \quad (2.3)$$

where $|+\rangle$ and $|-\rangle$ refer to an atom in the upper and lower state of the two-level, respectively, and the indexes 1 and 2 refer to the atom number.

In the Hamiltonian of (2.1), we distinguish between the coherent interaction H'' with the external applied field and the incoherent interaction H' with the continuum of the electromagnetic modes. The incoherent interaction describes the escape of excitation energy into the radiation field and is noted as a dissipative mechanism. This dissipation appears in a way that is analogous to a relaxation to a heat bath where the heat bath is understood as the quantum fluctuations of the photon vacuum.

The influence of the incoherent interaction H' on the equations of motion can be calculated separately from that of the coherent interaction in the following way. By assuming that the atomic system is in initial state $|\phi_m\rangle$ ($m = 1\cdots4$), its decay into a lower state is evaluated by developing the equations of motion due to H' and solving them by the use of the Laplace transform within the pole approximation (see explicit calculations in [19]).

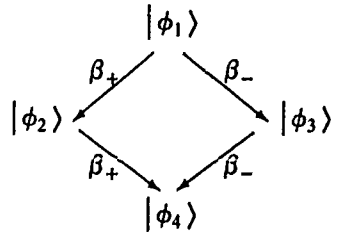
We find that the damping constants for the states $|\phi_2\rangle$ and $|\phi_3\rangle$ are given by β_+ and β_-, respectively, where

$$\beta_\pm = \beta(1 \pm g(r)) = \beta \pm B; \quad B = \beta g(r). \quad (2.4)$$

Here $\beta = 2\mu^2\omega^3/3\hbar c^3$ is one half of the Einstein A coefficient and $g(r)$ is a spatial factor which depends on the distance between the atoms. $g(r)$ is described explicitly in [19] or, in another form, $B = \beta g(r)$ is described in [21]. $g(r)$ tends to 1 for $r \to 0$ and tends to 0 for $r \to \infty$. The imaginary part of $\beta g(r) = B$ corresponds to frequency shifts and the absolute values of the amplitudes of $|\phi_2\rangle$ and $|\phi_3\rangle$ decay at a rate

$$\text{Re}\,\beta_\pm = \beta(1 \pm \text{Re}\,g(r)) = \beta \pm \Gamma \quad \Gamma. \quad (2.5)$$

In the same way, we find that the amplitude for the state $|\phi_1\rangle$ has a damping constant 2β where the decay constant from the state $|\phi_1\rangle$ to the state $|\phi_2\rangle$ is $\beta_+ = \beta(1 + g(r)) = \beta + B$, while that from $|\phi_1\rangle$ to $|\phi_3\rangle$ is $\beta_- = \beta(1 - g(r)) = \beta - B$. Taking into account the imaginary part of $g(r)$ (including frequency shifts), the effect of H' on the amplitudes of the states $|\phi_m\rangle$ can be described by the following diagram.



In the present work, we calculate the resonance fluorescence spectra in a strong field by including in our reduced system two sets of states: the states $|\phi_m\rangle$ ($m = 1\cdots4$) which do not include scattered photons, and the states $|\phi_{m,q}\rangle$ which include one scattered photon in the mode q. We introduce the atomic reduced density matrix elements

$$\rho_{mn} = \langle\phi_m|\rho_{at}|\phi_n\rangle; \quad \rho_{m,nq} = \langle\phi_m|\rho_{at}|\phi_{n,q}\rangle \quad (2.6)$$

where we have traced out the states of the continuum field. We calculate the equations of motion for these density matrix elements by using the Hamiltonian $H_0 + H''$ of (2.1). The effect of H' on the equations of motion has been reduced to the use of the damping constants evaluated previously for the amplitudes of the states $|\phi_m\rangle$ (or $\phi_{m,q}$). Under steady-state conditions, the calculation of the density matrix elements $\rho_{m,n}$ and $\rho_{m,nq}$ is reduced to the solution of algebraic equations. The procedure is completely analogous to the method used in [17], [18]. However, the calculations for our system of two, two-level atoms become much more complicated than those made for one two-level atom. The complication is mainly due to the fact that each of the sets $\rho_{m,n}$ and $\rho_{m,nq}$ include 16 matrix elements ($m = 1 \cdots 4$, $n = 1 \cdots 4$) and the solution of 16 simultaneous algebraic equations becomes quite tedious. Although in the present paper we develop the theory for general cases, we emphasize finally special cases for which the calculations and the physical solutions become quite simple.

The theory and the calculations which are presented here can be used further for evaluating the resonance fluorescence spectra of two-level atoms by numerical calculations solving the algebraic equations. In the following section, we present the calculations for the reduced density matrix elements $\rho_{m,n}$ and $\rho_{m,nq}$. These matrix elements are used for evaluating the resonance fluorescence spectra in Section IV.

III. The Density Matrix Equation of Motion

We introduce the slowly varying nondiagonal density matrix elements by using the definitions

$$\tilde{\rho}_{12} = \rho_{12}e^{i\omega_0 t} = \tilde{\rho}_{21}^*; \quad \tilde{\rho}_{24} = \rho_{24}e^{i\omega_0 t} = (\tilde{\rho}_{42})^*$$

$$\tilde{\rho}_{13} = \rho_{13}e^{i\omega_0 t} = (\tilde{\rho}_{31})^*; \quad \tilde{\rho}_{34} = \rho_{34}e^{i\omega_0 t} = (\tilde{\rho}_{43})^*$$

$$\tilde{\rho}_{14} = \rho_{14}e^{2i\omega_0 t} = (\tilde{\rho}_{41})^*; \quad \tilde{\rho}_{23} = \rho_{23} = (\tilde{\rho}_{32})^* \quad (3.1)$$

where the deviation from resonance is given by

$$\Delta = \omega - \omega_0. \quad (3.2)$$

We choose the phase of ω_R so that it is an imaginary number and define

$$\omega_R = i\alpha\sqrt{2} \quad (3.3)$$

where α is a real number. This choice of phase corresponds to the Hamiltonian of interaction and the interaction constant α defined in [18] for treating the resonance fluorescence spectra for one two-level atom.

In addition to the damping of the density matrix elements ρ_{mn}, we include in these equations the increase of the population of the levels $|\phi_2\rangle$, $|\phi_3\rangle$, and $|\phi_4\rangle$, represented by ρ_{22}, ρ_{33}, and ρ_{44}, due to the decay from the upper states. The equations for the diagonal matrix elements are dependent via the relation

$$\dot{\rho}_{11} + \dot{\rho}_{22} + \dot{\rho}_{33} + \dot{\rho}_{44} = 0 \quad (3.4)$$

so that we should eliminate one of the equations, e.g., for $\dot{\rho}_{22}$, and use instead the normalization condition

$$\rho_{11} + \rho_{22} + \rho_{33} + \rho_{44} = 1. \quad (3.5)$$

By assuming steady-state conditions, i.e., $\dot{\tilde{\rho}}_{mn} = 0$, we get 16 equations for the density matrix elements $\tilde{\rho}_{mn}$ ($m, n = 1, \cdots, 4$). All the density matrix elements, which are coupled to the antisymmetric state $|\phi_3\rangle$, can be eliminated from this equation in the following way.

From the equation

$$\dot{\rho}_{33} = 0 = 2(\beta - \mathrm{Re}\, B)\rho_{11} - 2(\beta - \mathrm{Re}\, B)\rho_{33} \quad (3.6)$$

we get the result

$$\rho_{33} = \rho_{11}. \quad (3.7)$$

The set of equations

$$\dot{\tilde{\rho}}_{13} = 0 = -i\Delta\tilde{\rho}_{13} - i\alpha\tilde{\rho}_{23} - (3\beta - B^*)\tilde{\rho}_{13}$$

$$\dot{\tilde{\rho}}_{23} = 0 = -i\alpha\tilde{\rho}_{13} - i\alpha\tilde{\rho}_{43} - 2(\beta - \mathrm{Im}\, B)\tilde{\rho}_{23}$$

$$\dot{\tilde{\rho}}_{34} = 0 = -i\Delta\tilde{\rho}_{34} + i\alpha\tilde{\rho}_{32} - (\beta - B)\tilde{\rho}_{34} \quad (3.8)$$

(including also their complex conjugates) are homogeneous algebraic equations whose only solutions are

$$\tilde{\rho}_{13} = \tilde{\rho}_{23} = \tilde{\rho}_{34} = 0. \quad (3.9)$$

The equations for the density matrix elements, which are not coupled to the state $|\phi_3\rangle$, become

$$\dot{\rho}_{11} = 0 = i\alpha(\tilde{\rho}_{12} - \tilde{\rho}_{21}) - 4\beta\rho_{11};$$

$$2\rho_{11} + \rho_{22} + \rho_{44} = 1$$

$$\dot{\rho}_{44} = 0 = i\alpha(\tilde{\rho}_{42} - \tilde{\rho}_{24}) + 2(\beta + \mathrm{Re}\, B)\rho_{22} + 2(\beta - \mathrm{Re}\, B)\rho_{33}$$

$$\dot{\rho}_{14} = 0 = -2i\Delta\tilde{\rho}_{14} + i\alpha(\tilde{\rho}_{12} - \tilde{\rho}_{24}) - 2\beta\tilde{\rho}_{14}$$

$$\dot{\tilde{\rho}}_{12} = -i\Delta\tilde{\rho}_{12} + i\alpha[\rho_{11} - \rho_{22} + \tilde{\rho}_{14}] - (3\beta + B^*)\tilde{\rho}_{12}$$

$$\dot{\tilde{\rho}}_{24} = 0 = -i\Delta\tilde{\rho}_{24} + i\alpha[\rho_{22} - \rho_{44} + \tilde{\rho}_{14}] - (\beta + B)\tilde{\rho}_{24} \quad (3.10)$$

where the equations for $\tilde{\rho}_{12}$, $\tilde{\rho}_{24}$, and $\tilde{\rho}_{14}$ are complex, while other equations are real. These equations may be easily calculated by numerical analysis.

For a very strong external field at exact resonance, the approximate solutions of (3.10) are given by

$$\rho_{11} \simeq \rho_{22} \simeq \rho_{33} \simeq \rho_{44} \simeq \frac{1}{4}; \quad \tilde{\rho}_{14} = 0$$

$$\tilde{\rho}_{21} \simeq \tilde{\rho}_{42} \simeq \frac{2i\beta\rho_{11}}{\alpha} \quad (\text{for } \Delta = 0; \ \alpha \gg \beta, \alpha \gg |B|). \quad (3.11)$$

Equations (3.11) represent the fact that when the pumping by the driving field at resonance ($\Delta = 0$) is very strong, the population in the four states becomes approximately equal. In the present paper, we discuss analytically the problem only for this limiting case. For other

cases, we represent the solutions by deriving sets of algebraic equations which can be solved numerically.

We introduce slowly varying density matrix elements $\rho_{m,nq}$ using the following definitions:

$$\tilde{\rho}_{2,1q} = \rho_{2,1q}e^{-2i\omega_0 t};\quad \tilde{\rho}_{1,2q} = \rho_{1,2q};\quad \tilde{\rho}_{4,1q} = \rho_{4,1q}e^{-3i\omega_0 t}$$

$$\tilde{\rho}_{4,2q} = \rho_{4,2q}e^{-2i\omega_0 t};\quad \tilde{\rho}_{2,4q} = \rho_{2,4q};\quad \tilde{\rho}_{4,4q} = \rho_{4,4q}e^{-i\omega_0 t}$$

$$\tilde{\rho}_{1,4q} = \rho_{1,4q}e^{i\omega_0 t};\quad \tilde{\rho}_{1,1q} = \rho_{1,1q}e^{-i\omega_0 t};\quad \tilde{\rho}_{2,2q} = \rho_{2,2q}e^{-i\omega_0 t}$$

$$\tilde{\rho}_{1,3q} = \rho_{1,3q};\quad \tilde{\rho}_{2,3q} = \rho_{2,3q}e^{-i\omega_0 t};\quad \tilde{\rho}_{4,3q} = \rho_{4,3q}e^{-2i\omega_0 t} \tag{3.12}$$

where ω_0 is the frequency of the external field. For these density matrix elements, we get under steady conditions, three independent sets of algebraic equations.

For $\tilde{\rho}_{m,nq}$ where $m \neq 3$, we get nine algebraic equations:

$$\dot{\tilde{\rho}}_{1,2q} = 0 = -\eta_q^*\rho_{11} + [-i(\Delta - \delta) - 3\beta - B^*]\tilde{\rho}_{1,2q} + i\alpha[\tilde{\rho}_{1,1q} - \tilde{\rho}_{2,2q} + \tilde{\rho}_{1,4q}]$$

$$\dot{\tilde{\rho}}_{4,1q} = 0 = [i(2\Delta + \delta) - 2B]\tilde{\rho}_{4,1q} + i\alpha[\tilde{\rho}_{4,2q} - \tilde{\rho}_{2,1q}]$$

$$\dot{\tilde{\rho}}_{2,1q} = 0 = [i(\Delta + \delta) - 3\beta - B]\tilde{\rho}_{2,1q} + i\alpha[\tilde{\rho}_{2,2q} - \tilde{\rho}_{1,1q} - \tilde{\rho}_{4,1q}]$$

$$\dot{\tilde{\rho}}_{2,2q} = 0 = [i\delta - 2(\beta + \mathrm{Re}\,B)]\tilde{\rho}_{2,2q} + i\alpha[\tilde{\rho}_{2,1q} + \tilde{\rho}_{2,4q} - \tilde{\rho}_{4,2q} - \tilde{\rho}_{1,2q}] + 2(\beta + \mathrm{Re}\,B)\tilde{\rho}_{1,1q} - \eta_q^*\rho_{21}$$

$$\dot{\tilde{\rho}}_{1,1q} = 0 = [i\delta - 4\beta]\tilde{\rho}_{1,1q} + i\alpha[\tilde{\rho}_{1,2q} - \tilde{\rho}_{2,1q}]$$

$$\dot{\tilde{\rho}}_{1,4q} = 0 = [-i(2\Delta - \delta) - 2\beta]\tilde{\rho}_{1,4q} + i\alpha[\tilde{\rho}_{1,2q} - \tilde{\rho}_{2,4q}] - \eta_q^*\tilde{\rho}_{12}$$

$$\dot{\tilde{\rho}}_{4,4q} = 0 = i\delta\tilde{\rho}_{4,4q} + i\alpha[\tilde{\rho}_{4,2q} - \tilde{\rho}_{2,4q}] - \eta_q^*\rho_{4,2} + 2(\beta - \mathrm{Re}\,B)\tilde{\rho}_{3,3q} + 2(\beta + \mathrm{Re}\,B)\tilde{\rho}_{2,2q}$$

$$\dot{\tilde{\rho}}_{2,4q} = 0 = [-i(\Delta - \delta) - (\beta + B)]\tilde{\rho}_{2,4q} + i\alpha[\tilde{\rho}_{2,2q} - \tilde{\rho}_{1,4q} - \tilde{\rho}_{4,4q}] - \eta_q^*\rho_{22}$$

$$\dot{\tilde{\rho}}_{4,2q} = 0 = [i(\Delta + \delta) - (\beta + B^*)]\tilde{\rho}_{4,2q} + i\alpha[\tilde{\rho}_{4,4q} + \tilde{\rho}_{4,1q} - \tilde{\rho}_{2,2q}] - \eta_q^*\rho_{41} \tag{3.13}$$

where

$$\delta = \omega_q - \omega_0,\ \Delta = \omega - \omega_0$$

and

$$\eta_q = \xi_q^{(1)}e^{i\vec{k}_q\cdot\vec{r}_1} + \xi_q^{(2)}e^{i\vec{k}_q\cdot\vec{r}_2}. \tag{3.14}$$

For the density matrix elements $\rho_{m,3q}$ where $m \neq 3$, we get three algebraic equations:

$$\dot{\tilde{\rho}}_{1,3q} = 0 = [-i(\Delta - \delta) - 3\beta + B^*]\tilde{\rho}_{1,3q} - i\alpha\tilde{\rho}_{2,3q} - \zeta_q^*\rho_{11}$$

$$\dot{\tilde{\rho}}_{2,3q} = 0 = [i\delta - 2(\beta + \mathrm{Im}\,B)]\tilde{\rho}_{2,3q} - i\alpha[\rho_{13,q} + \tilde{\rho}_{4,3q}] - \zeta_q^*\rho_{21}$$

$$\dot{\tilde{\rho}}_{4,3q} = 0 = [i(\Delta + \delta)\tilde{\rho}_{4,3q}] - (\beta - B^*)\rho_{4,3q} - \zeta_q^*\rho_{4,1} - i\alpha\rho_{2,3q} \tag{3.15}$$

where

$$\zeta_q = \xi_q^{(1)}e^{i\vec{k}_q\cdot\vec{r}_1} - \xi_q^{(2)}e^{i\vec{k}_q\cdot\vec{r}_2}. \tag{3.16}$$

For the density matrix elements $\rho_{3,nq}$ where $n = 1, \cdots, 4$, we get four algebraic equations:

$$\dot{\tilde{\rho}}_{3,1q} = 0 = [i(\Delta + \delta) - 3\beta + B]\tilde{\rho}_{3,1q} + i\alpha\tilde{\rho}_{3,2q}$$

$$\dot{\tilde{\rho}}_{3,2q} = 0 = [i\delta - 2(\beta - \mathrm{Im}\,B)]\tilde{\rho}_{3,2q} + i\alpha[\tilde{\rho}_{3,1q} + \tilde{\rho}_{3,4q}]$$

$$\dot{\tilde{\rho}}_{3,3q} = 0 = [i\delta - 2(\beta - \mathrm{Re}\,B)]\tilde{\rho}_{3,3q} \Rightarrow \tilde{\rho}_{3,3q} = 0$$

$$\dot{\tilde{\rho}}_{3,4q} = 0 = [-i(\Delta - \delta) - (\beta - B)]\tilde{\rho}_{3,4q} + i\alpha\tilde{\rho}_{3,2q} - \zeta_q^*\rho_{3,3}. \tag{3.17}$$

For the stationary state, we get $\tilde{\rho}_{3,3q} = 0$ so that (3.15) are independent of (3.17). For the general case, we have for $\tilde{\rho}_{m,nq}$ 15 algebraic equations which can be solved by simple numerical analysis (excluding $\tilde{\rho}_{3,3q} = 0$).

The use of the density matrix elements derived in the present section for calculating the resonance fluorescence spectra is described in the next section. We develop an analytical solution for the resonance fluorescence spectra for two, two-level atoms which are at resonance ($\Delta = 0$) and at a distance smaller than a wavelength.

IV. Resonance Fluorescence in a Strong Field

The resonance fluorescence spectra are given as

$$W_q = \frac{d}{dt}[\rho_{4q,4q} + \rho_{3q,3q} + \rho_{2q,2q}]_{\mathrm{spont}} \tag{4.1}$$

where only the growth due to the spontaneous emission (evaluated by the interaction H') can be seen by the detector, which is in a direction different from that of the applied field. In (4.1), we traced over the atomic states where we have taken into account that there is no spontaneous emission to the upper state $|\phi_1\rangle$.

By using the equation

$$\left[\frac{d\rho_{mq,mq}}{dt}\right]_{\mathrm{spont}} = -\frac{i}{\hbar}[H', \rho] \tag{4.2}$$

we get

$$\frac{d}{dt}[\rho_{2q,2q}]_{\mathrm{spont}} = -2\,\mathrm{Re}\,[\eta_q\rho_{1,2q}] \tag{4.3}$$

$$\frac{d}{dt}[\rho_{4q,4q}]_{\text{spont}} = -2\,\text{Re}\,[\eta_q\rho_{2,4q}] - 2\,\text{Re}\,[\zeta_q\rho_{3,4q}] \tag{4.4}$$

$$\frac{d}{dt}[\rho_{3q,3q}]_{\text{spont}} = -2\,\text{Re}\,[\zeta_q\rho_{1,3q}]. \tag{4.5}$$

The four terms on the right side of (4.2)–(4.4) correspond to the transitions described schematically in the diagram of Section II. In order to obtain the resonance fluorescence spectra, we need explicit expressions for the density matrix elements $\rho_{m,nq}$, which can be calculated by solving independent sets of algebraic equations given in the previous section as (3.10), (3.13), (3.15), and (3.17). Since the treatment of the general cases needs the use of very lengthy and tedious algebra, we prefer to analyze and discuss these cases by the use of numerical analysis in a separate paper. Here we limit the discussion to specific cases for which the theoretical analysis can be simplified and for which the physical results obtained by us can be compared to those of other investigators.

We investigate here the resonance fluorescence spectra for a very strong external field at exact resonance so that instead of using the first set of algebraic equations for $\rho_{m,n}$ given by (3.10), we use the approximate solutions given by (3.11). Even under this approximation, the analytical solutions of $\rho_{m,nq}$ are still very complicated and tedious. Therefore, we limit the discussion to the case for which $kr_{ij} \ll 1$. For this case, $B = \beta$, $e^{i\bar{k}\cdot\bar{r}_l} = e^{i\bar{k}\cdot\bar{r}_j} = 1$. Assuming that the two dipoles of the two atoms are equal, $\zeta_\xi = 0$ and (3.15) and (3.17) are homogeneous linear equations whose only solutions are

$$\tilde{\rho}_{1,3q} = \tilde{\rho}_{2,3q} = \tilde{\rho}_{4,3q} = \tilde{\rho}_{3,1q} = \tilde{\rho}_{3,2q} = \tilde{\rho}_{3,4q} = 0. \tag{4.6}$$

Therefore, the contributions to the resonance fluorescence spectra from the density matrix elements $\tilde{\rho}_{1,3q}$ and $\tilde{\rho}_{3,4q}$ of (4.4) and (4.5) vanish. This result is obvious as two atoms, which are within a distance smaller than a wavelength, are trapped in the state $|\phi_3\rangle$ with no transitions to other states.

We introduce the definitions

$$\frac{|\delta|}{\alpha} = \Omega = \frac{|\omega_q - \omega_0|}{\alpha};\quad \frac{\beta}{\alpha} = \epsilon. \tag{4.7}$$

We have developed an analytical expression for the line shape for the special case where the following approximations are valid:

$$kr_{ij} \ll 1;\quad \Delta = 0;\quad \epsilon \ll 1;\quad \frac{|\delta|}{\beta} = \frac{\Omega}{\epsilon} \gg 1. \tag{4.8}$$

The assumption of exact resonance ($\Delta = 0$) and the validity of the relation $B = \beta$ for $kr_{ij} \ll 1$ very much simplifies the expressions. The condition $\epsilon \ll 1$ refers to the approximation of a strong field where the Rabi line width $\alpha\sqrt{2}$ is large relative to the spontaneous decay line width β. For exact resonance, $|\delta| = |\omega_q - \omega|$ and the approximation $|\delta|/\beta \gg 1$ excludes the line center so that our approximations are valid for the line wings. The solution of the line shape under these approximations by following the present methods is interesting as it can be done analytically and the results can be compared to those of other investigators [7]–[13].

By following the above approximations, we have obtained solutions to $\rho_{m,nq}$ of (3.13), which are correct to first order in ϵ. By substituting these solutions in (4.1), (4.3), and (4.4), we get the following expression for the line shape:

$$W_q = \frac{|\eta_q|^2}{\alpha}\rho_{11}\left\{\frac{(\Omega^2-2)^2(6+4\Omega)\epsilon}{[\Omega(\Omega^2-2)]^2+\epsilon^2(8\Omega^2-6)^2} + \frac{2\Omega^2(\Omega^2-8)^2(3\Omega^2+4)\epsilon}{[\Omega(\Omega^2-2)(\Omega^2-8)]^2+\epsilon^2(14\Omega^4-82\Omega^2+48)^2}\right\}. \tag{4.9}$$

In Fig. 1, we describe the line shape W_q as a function of the normalized frequency $\Omega = |\omega_q - \omega_0|/\alpha$ for various values of $\epsilon = \beta/\alpha$. The line shape is given in relative units obtained by exact calculations of the expression in the curled brackets. The maximum of the line shape at $\Omega = \sqrt{2}$ corresponds to $\omega_q - \omega_0 = \alpha\sqrt{2}$ where $\alpha\sqrt{2}$ is the Rabi frequency. This maximum becomes narrower for small ϵ and its height is tending to ∞ for ϵ tending to zero. One can conclude this result analytically also as the second term of (4.9) becomes singular for $\epsilon \to 0$ at $\Omega = \sqrt{2}$. These results are not surprising as they are in agreement with the conclusions made by other investigators and the effect is similar to that obtained for one two-level atom. On the other hand, our conclusions about the additional side bands are different from those of other authors. Although our calculations show an additional maximum near $\Omega = 2\sqrt{2}$ (corresponding to twice the Rabi frequency: $2\sqrt{2}\alpha = |\omega_q - \omega_0|$), this maximum is quite broad and it does not show a singular behavior for ϵ tending to zero [7]. Taking into account the logarithmic scale used in Fig. 1, our conclusion is that the additional side bands are quite weak and are not in agreement with the relatively strong resonances predicted by other authors [7]–[10], [13].

We can explain our result by using a dressed state picture. The additional side bands result from transitions in which the dipole moment is doubled. The probability for this kind of transition for a stationary two, two-level atomic system is quite small, as the relative phases of different dipoles tend to be randomized. In a recent paper [12], we have suggested observing the additional side bands in a time dependent spectra where a strong transient cooperation will be induced by initial conditions.

V. Conclusions

The theoretical analysis made in the present paper for the resonance fluorescence spectra of driven two, two-level atoms can be used for evaluating the spectra in numerical calculations. The method is based on calculations of den-

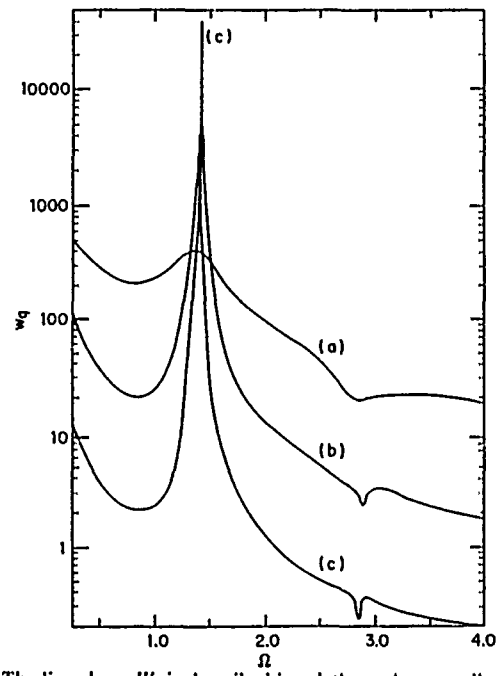


Fig. 1. The line shape W_q is described in relative units according to (4.9) as a function of the normalized frequency $\Omega = |\delta|/\alpha = |\omega_q - \omega_0|/\alpha$. The curves denoted by (a), (b), and (c) correspond to normalized spontaneous broadening $\epsilon = 0.1$, $\epsilon = 0.01$, and $\epsilon = 0.001$, respectively. This figure represents the line shape under the approximations given by (4.8).

sity matrix elements $\rho_{m,n}$ and $\rho_{m,nq}$ between atomic and atomic plus one photon states. We have presented the equations for the case in which the external radiation is propagating perpendicular to the interatomic axis, and have introduced the explicit spatial dependence of the coherent dipole-dipole interaction in the damping constants for the density matrix elements. The equations for $\rho_{m,n}$ and $\rho_{m,nq}$ are reduced by the present model to algebraic equations given, respectively, by (3.10) for $\rho_{m,n}$ and (3.13), (3.15), and (3.17) for $\rho_{m,nq}$. These density matrix elements are used in (4.1), (4.3), (4.4), and (4.5) for evaluating the resonance fluorescence spectra. For the case of two, two-level atoms at resonance interacting with a resonant external field, which are within a volume with a dimension smaller than a wavelength, we have derived an analytical expression for the line shape, presented in (4.9). We have obtained for this case side bands shifted from the line center by the Rabi frequency. Additional side bands appear, which are shifted from the line center approximately by twice the Rabi frequency. The additional side bands show very broad and weak maxima, and the result is in disagreement with the singular behavior predicted in previous works [7]–[10], [13].

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APPENDIX C

Quantum Fluctuations In Intrinsic Bistability of a Two-Level System

TECHNION-PH-89-25

QUANTUM FLUCTUATIONS IN INTRINSIC BISTABILITY OF A TWO LEVEL SYSTEM

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ABSTRACT

The quantum fluctuations in intrinsic bistability of a two-level system are calculated by using the Einstein relations. All the second moments of the dipole operators are calculated and the statistical properties of the reaction field are studied. We find that a quantum noise which is above the standard quantum limit is inherent in the reaction field.

1. Introduction

Intrinsic optical bistability (IOB) that does not require external optical feedback such as mirrors has been described in various theoretical and experimental works.[1–14] Most of the observations and the theoretical discussions have been made for complicated systems, e.g. in semiconductors, for which a full quantitative analysis is difficult. A relatively simple pure quantum mechanical system which produces IOB occurs for a dense collection of two-level atoms interacting via the electromagnetic field and driven by an external applied coherent field. For low values of the incident field the strong reaction field reduces the net field into a low transmitivity branch. As the incident field is increased at a certain critical intensity the cooperative interatomic interaction is broken and the contribution of the reaction field is suddenly negated. This phenomenon of a sudden transition from the low transmitivity branch into the high transmitivity branch occurs as a first order phase transition far from thermodynamic equilibrium. Although various properties of the coexisting steady states of the transmitted radiation have been described in our previous works, the influence of this phenomenon on photon statistics has not yet been explored.

The purpose of the present work is to study the quantum fluctuations in intrinsic bistability including especially the statistical properties of the reaction field.

In a recent work[15] we have treated quantum noise fluctuations in IOB two-lvel system. However, the analysis has been made only for a system with a rapid dephasing ($T_2 << T_1$) and only the affect of the quantum noise

on the bistable states has been discussed. In the present work we treat a pure quantum mechanical two-level system in which the relaxation time T_1 of the inversion of population is the spontaneous decay time and the relaxation time for the dipole moment is given by $T_2 = 2T_1$. By using the quantum theory of noise developed by Lax[16] we calculate all the second moments of the dipole operators and study the statistical properties of the quantum noise in a pure IOB system.

It has been shown by other authors that squeezing[17–19] can be produced by optical bistability[20] but this possibility has been studied only under the approximation of "a good cavity limit", i.e. for bistability in high Q cavity[21]. For such cases the medium relaxation is much faster than that of the field and then the atomic variabls can be adiabatically eliminated. We investigate here the problem of photon statistics for the extreme case of a "bad cavity limit"[21] where we preclude any reflecting boundaries and where the relaxation of the field in the medium is much faster than the relaxation of the material. For this case the statistical properties of the reaction field are related to the statistical properties of the dipole operators.

A single atom driven by an externally applied coherent field emits spontaneous photons that are "antibunched" i.e. the intensity correlation function is in the quantum domain $g^{(2)}(0) < 1$ for a null delay.[22–23]. This nonclassical property of the light observed in resonance fluorescence of single atoms occurs due to the reduction of the state of the atom into its ground state after the emission of the first photon so that it cannot immediately emit a second photon. The antibunching phenomena vanish for the radiation emitted in

resonance fluorescence of many two-level atoms due to destructive interference effects. We will show in a similar way that the quantum noise is inherent in the IOB system and that squeezing[17–19] cannot be obtained in this system (the bad cavity limit).

In a recent work antibunching behaviour has been observed in the resonance fluorescence light emitted by a many atom source.[24] The constructive phase matching conditions in that work were obtained by using a four wave mixing scheme in which the intensity correlation function was measured between two detection channels of spontaneous radiation emitted in opposite directions. Although this system is basically different from that of the present work the methods developed here can be used also for treating that problem and we hope to do it somewhere else.

In Sec. 2 we describe shortly some properties of Squeezed States (SS) and explain our method for checking the existence of squeezing. In Sec. 3 we calculate all the moments of the dipole operators in IOB by using Einstein relations[16,25]. The statistical properties of the transmitted radiation are discussed and we find that squeezing is not obtained in the present system. In Sec. 4 we summarize our results and conclusions.

2. Squeezing and Anomalous Correlation Functions

The nonclassical properties of quantum light[26] are related to the normal ordered normalized second order correlation function defined for stationary

light as

$$g^{(2)}(\tau) = \frac{< E^{(-)}(t)E^{(-)}(t+\tau)E^{(+)}(t+\tau)E^{(+)}(t) >}{< E^{(-)}(t)E^{(+)}(t) >^2}\ , \qquad (1)$$

where $E^{(+)}$ and $E^{(-)}$ are the positive-frequency and negative-frequency parts of the elctromagnetic field. The non-classical properties of S.S. are usually obtained by mixing the L.O. (coherent state) and the S.S. in interference experiments.[27-28] It is generally assumed in these experiments that the L.O. is much stronger than the S.S. so that the denominator of Eq. (1) is equal approximately to $|\alpha|^4$ where $|\alpha|$ is the absolute value of the coherent state amplitude. The numerator of Eq. (1) can be developed up to the second order in the S.S. operators and the result is given by[29-31]

$$g^{(2)}(\tau) - 1 = \frac{<: \Delta[E_s^{(\theta)}(t)E_s^{(\theta)}(t+\tau)] :>}{|\alpha|^2} \qquad (2)$$

where :: denotes normal ordering and $E_s^{(\theta)}$ is the quadrature amplitude of the S.S.:[32]

$$E_s^{(\theta)}(t) = E_s^{(-)}(t)e^{i\theta} + E_s^{(+)}(t)e^{-i\theta} \qquad (3)$$

θ is the phase angle of the coherent state which is changed in the interference experiment to obtain an optimal quantum noise reduction. The quadrature amplitudes are defined in the interaction picture of the L.O. so that the rapid dependence on the frequency of the L.O. is eliminated. The function $g^{(2)}(\tau)$ is dimensionless and since the normalization is arbitrary it is chosen so that $|\alpha|^2$ is the flux of the L.O. photons (sec^{-1}) and consistently $<: \Delta[E_s^{(\theta)}(t)E_s^{(\theta)}(t+\tau)] :>$ is given also in terms of the flux of S.S. photons.

Non-classical light is defined by the antibunching property $g^{(2)}(\tau) < 1$.[26] Similar expressions to Eq. (2) have been used extensively in various works for analyzing quantum noise reduction under the Shot Noise Limit (SNL) in homodyne and heterodyne experiments[19,29-31] or explaining other quantum effects in correlation measurements between two photodetectors[33]. In particular the Fourier transform of Eq. (2) is known as the squeezing spectrum and various theoretical methods have been developed for calculating this spectrum.[32,34-36]

The anomalous correlation function for the S.S. is defined as[35]

$$J_{++}(\tau) = <: \Delta[E_s^{(+)}(t)E_s^{(+)}(t+\tau)] :> = |J_{++}(\tau)|e^{-i\phi} \tag{4}$$

and the ordinary correlation function as[35]

$$J_{-+}(\tau) = <: \Delta[E_s^{(-)}(t)E_s^{(+)}(t+\tau)] :> \tag{5}$$

By using these definitions Eq. (2) can be expressed as

$$g^{(2)}(\tau) - 1 = cos^2(\theta+\phi/2)[2|J_{++}(\tau)| + J_{-+}(\tau) + J_{+-}(\tau)] \tag{6}$$
$$+sin^2(\theta+\phi/2)[-2|J_{++}(\tau)| + J_{-+}(\tau) + J_{+-}(\tau)],$$

where $[J_{++}(\tau)]^* = J_{--}(\tau)$. It will be shown later that $J_{-+}(\tau)$ and $J_{+-}(\tau)$ are for IOB real positive functions of τ. Since $g^{(2)}(\tau) - 1$ expresses the amount of quantum noise this noise is minimum for $sin^2(\theta+\phi/2) = 1$ and maximum for $cos^2(\theta+\phi/2) = 1$ with harmonic dependence on the L.O. phase θ.

In the present work we are interested in the statistical properties of the reaction field in IOB. It is sufficient to calculate $g^{(2)}(0)$ in order to see if antibunching effect ($g^{(2)}(0) < 1$) occurs in the system which would, in principle,

lead to squeezing spectra. The condition $g^{(2)}(0) < 1$ gives according to Eq. (6) the following criterion for obtaining squeezed spectra

$$2|J_{++}(0)| > J_{-+}(0) + J_{+-}(0) \tag{7}$$

for null delay times. Squeezing is obtained when the relation (7) is fulfilled since by adjusting the phase θ of the L.O. we get the non-classical regime $g^{(2)}(0) < 1$ and the quantum noise is reduced below the SNL in the homodyne or heterodyne or other correlation experiments. The meaning of inequality (7) is that for obtaining squeezing the anomalous correlation function of creating or annihilating two photons should be larger than that of the usual correlation function of creating or annihilating one photon.

The role of anomalous correlation functions in relation to squeezing has been treated in various works.[31,35,37] In particular the constructive anti-bunching effects described in Ref. 24 were obtained by designing the experiments so that constructive interference is obtained for the anomalous correlation function $J_{++}(\tau)$ and a destructive interference is produced for the ordinary correlation functions $J_{+-}(\tau)$ and $J_{-+}(\tau)$.

By using definitions analogeous to Eqs. (4-5) and the inequality (7) the criterion for obtaining squeezing in IOB is

$$2| < \delta\sigma_{+0}\delta\sigma_{+0} > | \; > \; < \delta\sigma_{-0}\delta\sigma_{+0} > + < \delta\sigma_{+0}\delta\sigma_{-0} > . \tag{8}$$

We have chosen to treat in the present work a pure quantum system since as is well known the insertion of additional damping mechanism spoil the effects of squeezing. However, we will show that even in the pure quantum

mechanical system a quantum noise exists which is above the Quantum Noise Limit (QNL). We calculate in the second section all the second moments of the dipole operators and examine the statistical properties of the reaction field.

3. Statistical Properties of the Quantum Noise in IOB

We consider a thin sample of a large number of two-level atoms coupled to each other only via the electromagnetic field and driven externally by an applied coherent field. In our previous works on IOB in two-level systems we have calculated for various cases the expectation values per unit volume of the slowly varying dipole moments $< \sigma_{\pm 0} >$ and for the inversion of population $< \sigma_z >$.[12-14] In the calculations for the expectation values the contribution of the Langevin noise terms vanish. In the present work we assume the relations

$$\hat{\sigma}_{\pm 0} =< \sigma_{\pm 0} > +\delta\hat{\sigma}_{\pm 0}; \quad \hat{\sigma}_z =< \sigma_z > +\delta\hat{\sigma}_z \tag{9}$$

and linearize the equations for small perturbations of the system from the stationary state. By following the derivations given in our previous works[12-14] including the Langevin noise terms we get

$$\begin{aligned} \frac{d(\delta\sigma_{+0})}{d\tau} &= [-\beta/2 + i(\Delta - \epsilon < \sigma_z >]\delta\hat{\sigma}_{+0} \\ &+ [\omega_R^*/2 - i\epsilon < \sigma_{+0} >]\delta\hat{\sigma}_z + f^{(-)} < \sigma_z > \ ; \\ \frac{d(\delta\sigma_z)}{d\tau} &= -\beta\delta\sigma_z - \omega_R\delta\hat{\sigma}_{+0} - \omega_R^*\delta\hat{\sigma}_{-0} - 2f^{(+)}\langle\hat{\sigma}_{+0}\rangle - 2f^{(-)} < \sigma_{-0} > \end{aligned} \tag{10}$$

We neglect here the multiplications of $f^{(+)}$, $f^{(-)}$ with $\delta\sigma_z, \delta\sigma_{\pm 0}$. The present approximations are not valid when the quantum fluctuations are large or when the system is unstable.

We assume a fully quantum mechanical model where $\beta = 4|P|^2k^3/3k$ is spontaneous decay time. P is the dipole matrix element and $k = \omega/c$. $\Delta = \omega - \omega_0$ is the detuning, where ω is the frequency of the applied field E, and ω_0 is the atomic resonance frequency. ω_R is the Rabi rate associated with the applied coherent field and ϵ is the frequency renormalization constant derived for a dense two-level system in our previous work as

$$\epsilon = \frac{7\pi n\beta c^3}{4\omega_0^2} \,. \tag{11}$$

The frequency renormalization stems from coherent dipole dipole interaction between atoms that are within a volume of a cubic wavelength. The free field operators of Eqs. (10) obey in the plane wave limit the equations:

$$< f^{(+)}(\tau)f^{(-)}(\tau') >= \beta\delta(\tau - \tau'),$$
$$< f^{(+)}(\tau)f^{(+)}(\tau') >=< f^{(-)}(\tau)f^{(-)}(\tau') >=< f^{(-)}(\tau)f^{(+)}(\tau') >= 0 \tag{12}$$

where τ is the retarded time. These operators constitute fluctuating noise terms which stem from the quantum field coupling to atoms.

We follow in our theoretical treatment of the quantum noise in IOB the quantum theory of noise developed by Lax[16]. We rewrite our fundamental Langevin Eqs. (10) in the form:

$$\frac{da_\mu}{dt} = G_\mu(\vec{a}, t) + F_\mu(\vec{a}, t) \tag{13}$$

where for our cases $\vec{a}, \vec{G}$ and $\vec{F}$ are three dimensional vectors given by:

$$a_1 = \delta\hat{\sigma}_{+0}; \quad a_2 = \delta\hat{\sigma}_{-0}; \quad a_3 = \delta\hat{\sigma}_z \tag{14}$$

$$\begin{aligned} G_1 &= [-\beta/2 + i(\Delta - \epsilon < \sigma_z >)]\delta\hat{\sigma}_{+0} + [\omega_R^*/2 - i\epsilon < \sigma_{+0} >]\delta\hat{\sigma}_z, \\ G_2 &= G_1^+; \; G_3 = -\beta\delta\hat{\sigma}_z - \omega_R\delta\hat{\sigma}_{+0} - \omega_R^*\delta\hat{\sigma}_{-0} \; , \end{aligned} \tag{15}$$

and

$$F_1 = f^{(-)} < \hat{\sigma}_z >; \; F_2 = f^{(+)} < \sigma_z >; \; F_3 = -2f^{(+)} < \sigma_{+0} > -2f^{(-)} < \sigma_{-0} > \tag{16}$$

$F_\mu(t)$ are Markoffian operators and according to Eq. (12) obey the equations

$$< F_\mu(t)F_\nu(u) >= 2 < D_{\mu\nu}(\vec{a}, t) > \delta(t-u) \tag{17}$$

where

$$\begin{aligned} &< D_{11} >=< D_{22} >=< D_{12} >=< D_{13} >=< D_{32} >= 0; \\ &2 < D_{23} >= -2\beta < \sigma_z >< \sigma_{-0} >; \; 2 < D_{21} >= \beta < \sigma_z >^2, \\ &2 < D_{31} >= -2\beta < \sigma_{+0} >< \sigma_z >; \; 2 < D_{33} >= 4 < \sigma_{+0} >< \sigma_{-0} > \beta \end{aligned} \tag{18}$$

One should notice that $< D_{\mu\nu} >$ is not symmetric as F_μ and F_ν do not commute.

By using the Markoffian property of the operator $F_\mu(t)$ Lax has shown[16] that the equation for the mean motion of the product of two operators a_μ and a_ν obeying Eq. (13) is given by:

$$\frac{d < a_\mu a_\nu >}{dt} = 2 < D_{\mu\nu} > + < G_u a_\nu > + < a_\nu G_\mu > \; . \tag{19}$$

In the steady state when the second order operators do not change with time we obtain the standard Einstein relations

$$-2 < D_{\mu\nu} >_{\text{steady state}} = < G_\mu a_\nu >_{ss} + < a_\mu G_\nu >_{ss} \ . \qquad (20)$$

The order in these equations follows from the quantum nature of our system and in this respect it is different from the equations for the moments in a Markoffian system which can be derived classically.

We introduce the following definitions:

$$p = -\beta/2 + i[\Delta - \epsilon < \sigma_z >]; \quad q = \omega_R^*/2 - i\epsilon < \sigma_{+0} > \ . \qquad (21)$$

Einstein equations for our system are obtained from Eqs. (20) by substituting the values of a_μ, G_ν and $D_{\mu\nu}$ according to Eqs. (14), (15) and (18), respectively, and by using the short notation of Eq. (21).

We get the following 9 Einstein equations for our system:

$$< \delta\sigma_z \delta\sigma_{+0} > + < \delta\sigma_{+0} \delta\sigma_z > = -(2p/q) < \delta\sigma_{+0} \delta\sigma_{+0} >;$$

$$< \delta\sigma_z \delta\sigma_{-0} > + < \delta\sigma_{-0} \delta\sigma_z > = -(2p^*/q^*) < \delta\sigma_{-0} \delta\sigma_{-0} >;$$

$$< \delta\sigma_z^2 > = 2 < \sigma_{+0} >< \sigma_{-0} > + \frac{\omega_R p}{\beta q} < \delta\sigma_{+0} \delta\sigma_{+0} >$$
$$+ \frac{\omega_R^* p^*}{\beta q^*} < \delta\sigma_{-0} \delta\sigma_{-0} > ;$$

$$q < \delta\sigma_z \delta\sigma_{-0} > + q^* < \delta\sigma_{+0} \delta\sigma_z > = \beta < \delta\sigma_{+0} \delta\sigma_{-0} >;$$

$$(\rho-\beta)<\delta\sigma_{+0}\delta\sigma_z>+q<\delta\sigma_z\delta\sigma_z>=\omega_R<\delta\sigma_{+0}\delta\sigma_{+0}>$$
$$+\omega_R^*<\delta\sigma_{+0}\delta\sigma_{-0}>;$$

$$(p^*-\beta)<\delta\sigma_{-0}\delta\sigma_z>+q^*<\delta\sigma_z\delta\sigma_z>=\omega_R^*<\delta\sigma_{-0}\delta\sigma_{-0}>$$
$$+\omega_R<\delta\sigma_{-0}\delta\sigma_{+0}>+2\beta<\sigma_z><\sigma_{-0}>;$$

$$q<\delta\sigma_{-0}\delta\sigma_z>+q^*<\delta\sigma_z\delta\sigma_{+0}>=-\beta<\sigma_z^2>+\beta<\delta\sigma_{-0}\delta\sigma_{+0}>;$$

$$(p-\beta)<\delta\sigma_z\delta\sigma_{+0}>+q<\delta\sigma_z^2>-\omega_R<\delta\sigma_{+0}\delta\sigma_{+0}>$$
$$-\omega_R^*<\delta\sigma_{-0}\delta\sigma_{+0}>=2\beta<\sigma_{+0}><\sigma_z>;$$

$$(p^*-\beta)<\delta\sigma_z\delta\sigma_{-0}>+q^*<\delta\sigma_z^2>-\omega_R<\delta\sigma_{+0}\delta\sigma_{-0}>$$
$$-\omega_R^*<\delta\sigma_{-0}\delta\sigma_{-0}>=0 \qquad (22)$$

Eqs. (22) consist of 9 algebraic equations from which one can calculate numerically all the moments for our system. A careful analysis of these equations shows that they fulfill the following relations:

$$<\delta\sigma_z\delta\sigma_{+0}>^*=<\delta\sigma_{-0}\delta\sigma_z>;\ <\delta\sigma_z\delta\sigma_{-0}>^*=<\delta\sigma_{+0}\delta\sigma_z>$$
$$<\delta\sigma_{+0}\delta\sigma_{+0}>^*=<\delta\sigma_{-0}\delta\sigma_{-0}> \qquad (23)$$

The functions $<\delta\sigma_z^2>$, $<\delta\sigma_{+0}\delta\sigma_{-0}>$ and $<\delta\sigma_{-0}\delta\sigma_{+0}>$ are real and positive but $<\delta\sigma_{+0}\delta\sigma_{-0}>\neq<\delta\sigma_{-0}\delta\sigma_{+0}>$ due to the unsymmetric property of $D_{\mu\nu}$.

By using the above relations, Eqs. (22) can be reduced to the following form

$$< \{\delta\sigma_z, \delta\sigma_{+0}\} >= -\frac{2p}{q} < \delta\sigma_{+0}\delta\sigma_{+0} > \qquad (24\text{-}a)$$

$$< \delta\sigma_z^2 >= 2 < \sigma_{+0} >< \sigma_{-0} > +(\frac{\omega_R p}{\beta q} < \delta\sigma_{+0}\delta\sigma_{+0} > +c.c.); \qquad (24\text{-}b)$$

$$(q^* < \{\delta\sigma_z, \delta\sigma_{+0}\} > +c.c.) = - < \sigma_z >^2 \beta + \beta < \{\delta\sigma_{+0}, \delta\sigma_{-0}\} >; \qquad (24\text{-}c)$$

$$(q^*[\delta\sigma_z, \delta\sigma_{+0}] + c.c.) = -\beta < \sigma_z >^2 + \beta < [\delta\sigma_{-0}, \delta\sigma_{+0}] >; \qquad (24\text{-}d)$$

$$(p-\beta) < \{\delta\sigma_z, \delta\sigma_{+0}\} > +2q < \delta\sigma_z^2 >=$$
$$2\omega_R < \delta\sigma_{+0}\delta\sigma_{+0} > +\omega_R^* \{\delta\sigma_{+0}, \delta\sigma_{-0}\} + 2\beta < \sigma_{+0} >< \sigma_z >; \qquad (24\text{-}e)$$

$$(p-\beta) < [\delta\sigma_z, \delta\sigma_{+0}] >= \omega_R^*[\delta\sigma_{-0}, \delta\sigma_{+0}] + 2\beta < \sigma_{+0} >< \sigma_z >; \qquad (24\text{-}f)$$

where in these equations { } and [ ] denote anticommutation and commutation relations respectively.

In the present work we are treating the fluctuations in the reaction field and check the possibility for getting squeezed light according to Eq. (8). For this purpose it is enough to calculate the moments $< \delta\sigma_{+0}\delta\sigma_{+0} >=< \delta\sigma_{-0}\delta\sigma_{-0} >^*$ and $< \delta\sigma_{+0}\delta\sigma_{-0} > + < \delta\sigma_{-0}\delta\sigma_{+0} >= \{\delta\sigma_{+0}, \delta\sigma_{-0}\}$. By substituting Eq. (24a) into Eq. (24c) we get:

$$< \{\delta\sigma_+, \delta\sigma_{-0}\} > +\frac{2pq^*}{\beta q} < \delta\sigma_{+0}\delta\sigma_{+0} > +\frac{2p^*q}{\beta q^*} < \delta\sigma_{-0}\delta\sigma_{-0} >=< \sigma_z >^2 \qquad (25)$$

By substituting Eqs. (24-a) and (24-b) into Eq. (24-e) we get

$$< \{\delta\sigma_{+0}, \delta\sigma_{-0} > + < \delta\sigma_{+0}\delta\sigma_{+0} > [\frac{2\omega_R}{\omega_R^*} + \frac{2p(p-\beta)}{q\omega_R^*} - \frac{2p\omega_R}{\beta\omega_R^*}]$$
$$- < \delta\sigma_{-0}\delta\sigma_{-0} > \frac{2qp^*}{q^*\beta} = \frac{4q < \sigma_{+0} >< \sigma_{-0} >}{\omega_R^*} - \frac{2\beta < \sigma_{+0} >< \sigma_z >}{\omega_R^*} \quad (26)$$

We find that the moments needed for calculating the properties of the reaction field can be calculated from Eqs. (25) and (26) without the need to evaluate ail other moments. Let us introduce the following definitions

$$< \delta\sigma_{+0}\delta\sigma_{+0} > \frac{\omega_R}{\omega_R^*} = A; \quad < \{\delta\sigma_{+0}, \delta\sigma_{-0}\} >= 2r; \quad (27)$$

where r is real and positive and the condition for squeezing becomes $r < |A|$.

For the steady state solutions of IOB the following relations are fulfilled

$$< \sigma_{+0} >= \frac{-(\omega_R^*/2) < \sigma_z >}{i(\Delta - \epsilon < \sigma_z >) - (\beta/2)},$$
$$< \sigma_{+0} >< \sigma_{-0} >= \frac{- < \sigma_z > [< \sigma_z > +1]}{2} \quad (28)$$

Using these relations the real and imaginary parts of Eq. (26), and Eq. (25) consist of three algebraic equations from which the parameters r and A can be calculated and the condition $r < |A|$ for squeezing can be examined by the numerical calculations. The phase of ω_R/ω_R^*, fixed by an arbitrary phase in the external field, will affect the phase of $< \delta\sigma_{+0}\delta\sigma_{+0} >$ but will not change the results for the values of r and $|A| = | < \delta\sigma_{+0}\delta\sigma_{+0} > |$.

By substituting the value of $< \sigma_{+0} >$ from Eqs. (28) into Eq. (21) we get:

$$q = (\omega_R^*/2)[\frac{(i\Delta - \beta/2)}{i(\Delta - \epsilon < \sigma_z >) - \beta/2}] \quad (29)$$

Since general analytical solutions of Eqs. (25-26) are still quite complicated we pursue further our quantitative analysis for three special cases and obtain qualitatively general conclusions.

Let us develop the analytical solutions under the resonant condition $\Delta = 0$ where we get:

$$q = (\omega_R^*/2)\frac{\beta}{\beta + 2i\epsilon < \sigma_z >}; \quad p = -\beta/2 - i\epsilon < \sigma_z > \tag{30}$$

Substituting Eqs. (27), (28) and (30) into Eqs. (25-26) we get

$$2r - A\frac{(\beta + 2i\epsilon < \sigma_z >)^2}{\beta(\beta - 2i\epsilon < \sigma_z >)} - A^*\frac{(\beta - 2i\epsilon < \sigma_z >)^2}{\beta + 2i\epsilon < \sigma_z >} =< \sigma_z >^2 \tag{31}$$

$$2r + A[2 + \frac{\beta + 2i\epsilon < \sigma_z >}{\beta} + \frac{(\beta + 2i\epsilon < \sigma_z >)^2(3\beta + 2i\epsilon < \sigma_z >)}{|\omega_R|^2\beta}],$$
$$+A^*\frac{(\beta - 2i\epsilon < \sigma_z >)^2}{\beta(\beta + 2i\epsilon < \sigma_z >)} = \frac{-\beta}{\beta + 2i\epsilon < \sigma_z >}[3 < \sigma_z >^2 + < \sigma_z >]. \tag{32}$$

We study the quantum fluctuations in the reaction field both for the ordinary Bloch equations ($\epsilon = 0$) and for the IOB case ($\epsilon >> \beta$).

For the ordinary Bloch equations on resonance we have

$$\epsilon = 0; \quad \beta^2/|\omega_R|^2 = -2 < \sigma_z > /[< \sigma_z > +1] \tag{33}$$

and Eqs. (31-32) can be written as

$$2r - (A + A^*) =< \sigma_z >^2, \tag{34}$$

$$2r + 3A[1 + \frac{\beta^2}{|\omega_R|^2}] + A^* = -3 < \sigma_z >^2 - < \sigma_z > \tag{35}$$

After a straightforward algebra we get

$$A_I \equiv ImA = 0;\ A_R \equiv ReA = -\frac{2}{3} < \sigma_z >^2 [< \sigma_z > +1] - \frac{\sigma_z[< \sigma_z > +1]}{6};$$
$$r = ReA + < \sigma_z >^2 /2. \tag{36}$$

In Fig. 1 we describe the values of r and $|A|$ as a function of $< \sigma_z >$ according to Eqs. (36) for Bloch equations on resonance. We find that $r > |A|$ so that the reaction field is bunched and squeezing cannot be obtained for the transmitted radiation. We find also the conclusion that the fluctuations in the ordinary correlation function are decreasing as a function of the intensity of the external field so that they are negligible in the region of $< \sigma_z > \simeq 0$ and maximal in the region $< \sigma_z > \simeq -1$.

Let us now discuss the solutions of Eqs. (32-33) for resonance ($\Delta = 0$) in intrinsic bistability ($\epsilon >> \beta$). For this case the relation

$$|\omega_R|^2 = -\frac{[< \sigma_z > +1][\beta^2 + 4\epsilon^2 < \sigma_z >^2]}{2 < \sigma_z >} \tag{37}$$

is fulfilled. We simplify further Eqs. (32-33) under the approximation

$$2\epsilon | < \sigma_z > | >> \beta\ . \tag{38}$$

This approximation is very good for the lower branch of intrinsic bistability where we have relatively stronger quantum fluctuations. We get under this approximation:

$$2r + 6A_R - \frac{4\epsilon < \sigma_z >}{\beta} A_I = < \sigma_z >^2 \tag{39}$$

$$2r + A_R \frac{10 < \sigma_z >}{< \sigma_z > +1} - A_I \frac{4\epsilon < \sigma_z >^2}{\beta[< \sigma_z > +1]} = \frac{-\beta^2}{4\epsilon^2 < \sigma_z >^2} [3 < \sigma_z >^2 + \sigma_z >] \tag{40}$$

$$A_I[6+\frac{10<\sigma_z>}{<\sigma_z>+1}]+A_R[\frac{4\epsilon<\sigma_z>}{\beta}+\frac{4\epsilon<\sigma_z>^2}{\beta[<\sigma_z>+1]}]$$
$$=\frac{\beta}{2\epsilon<\sigma_z>}[3<\sigma_z>^2+<\sigma_z>]\,. \tag{41}$$

In Fig. 2 the values of r and $|A|$ are calculated, as a function of $|<\sigma_z>| \geq 0.5$ according to Eqs. (39-41) for an IOB example on resonance ($\Delta = 0$) for which $\epsilon = 10\beta$ and the condition (38) is valid. We find that the quantum fluctuations for an IOB system are similar to those of the ordinary Bloch equations. Similar to the ordinary Bloch equations $r > |A|$ so that squeezing is not obtained. For the upper branch in IOB the terms which are proportional to $\epsilon <\sigma_z>$ are quite small so that the equations become similar to the ordinary Bloch equations where squeezing is not obtained.

Let us now discuss as a third case the solutions of Eqs. (25-26) for IOB under the resonant condition $\Delta - \epsilon <\sigma_z> = 0$ where the relation

$$\beta^2/|\omega_R|^2 = -2<\sigma_z>/[<\sigma_z>+1] \tag{42}$$

is valid (as in Eq. (33)). For this case the relations

$$p=-\beta/2;\ <\sigma_{+0}>=\frac{\omega_R^*<\sigma_z>}{\beta};\ q=(\omega_R^*/2)[\frac{\beta-2i\epsilon<\sigma_z>}{\beta}] \tag{43}$$

are fulfilled. By using Eqs. (43) and Eqs. (27) we can write Eqs. (25-26) as

$$2r-[A\frac{\beta+2i\epsilon<\sigma_z>}{\beta-2i\epsilon<\sigma_z>}+c.c.]=<\sigma_z>^2\,, \tag{44}$$

$$2r+A[3+\frac{3\beta^2}{|\omega_R|^2}\frac{\beta}{\beta-2i\epsilon<\sigma_z>}]+A^*\frac{\beta-2i\epsilon<\sigma_z>}{\beta+2i\epsilon<\sigma_z>}=$$
$$\frac{2|\omega_R|^2<\sigma_z>^2}{\beta^3}(\beta-2i\epsilon<\sigma_z>)-2<\sigma_z>^2 \tag{45}$$

Using the relation (42) and the approximation (38) we reduce Eqs. (44-45) into the form:

$$2r + 2A_R + A_I \frac{\beta}{\epsilon < \sigma_z >} = < \sigma_z >^2, \tag{46}$$

$$2r + A_R[2 - \frac{3\beta^2}{[< \sigma_z > +1]2\epsilon^2 < \sigma_z >}] + A_I[\frac{3\beta}{\epsilon[< \sigma_z > +1]} - \frac{\beta}{\epsilon < \sigma_z >}]$$
$$= -2 < \sigma_z >^2 - < \sigma_z > [< \sigma_z > +1] \ , \tag{47}$$

$$A_I[4 - \frac{3\beta^2}{[< \sigma_z > +1]2\epsilon^2 < \sigma_z >}] - A_R[\frac{\beta}{\epsilon < \sigma_z >} + \frac{3\beta}{[< \sigma_z > +1]\epsilon}]$$
$$= \frac{2 < \sigma_z >^2 \epsilon[< \sigma_z > +1]}{\beta} \tag{48}$$

In Fig. 3 the values of r and $|A|$ are calculated as a function of $| < \sigma_z > | \geq 0.5$ according to Eqs. (46-48) for an IOB example on the resonance $\Delta - \epsilon < \sigma_z >= 0$ for which $\epsilon = 10\beta$ and the condition (38) is valid. We find that the quantum fluctuations in this case are very large relative to those in Fig. 2. Squeezing is not obtained in this example and the reaction field is again bunched. Although we evaluated the parameters A and r according to Eqs. (25-26) under special conditions we can obtain general conclusions. Under the resonance condition $\Delta - \epsilon < \sigma_z >= 0$ the absolute value of the parameter $\frac{2pq^*}{\beta q}$ in Eq. (25) is minimal. On deviation from this resonance we would expect therefore a decrease in the parameter $|< \delta\sigma_{+0}\delta\sigma_{+0} >|$, i.e. worse conditions for squeezing, We summarize our conclusions in the next section.

4. Results and Conclusions

The statistical properties of the reaction field in IOB has been studied. The quantum properties of the reaction field are related to the anomalous correlation function and a general criterion for obtaining squeezing is given [Eq. (8)].

General equations for calculating all the second moments of the dipole operators have been developed by linearizing the equations for small perturbations of the system from the stationary state and by using the corresponding Einstein relations [Eqs. (22-24)]. The fluctuations in the reaction field can be calculated numerically from Eqs. (25-27) where the condition $2\,|<\delta\sigma_{+0}\delta\sigma_{+0}>|> <\delta\sigma_{+0}\delta\sigma_{-0}> + <\delta\sigma_{-0}\delta\sigma_{+0}>$ represent the antibunching property (and vice versa).

By solving our equations for special examples and examining the general behaviour of the equations we find that the reaction field is bunched so that a quantum noise which is above the quantum noise limit is inherent in the IOB system. We may conclude therefore that the photon statistics in the bad cavity limit is more chaotic than that of the good cavity limit in which squeezing was predicted. We find however the conclusion that the quantum fluctuations in the steady states of IOB on resonance ($\Delta = 0$) are similar to those predicted for the ordinary bloch equations ($\epsilon = 0$) on resonance for the same state of inversion of population.

The calculations made in this work show that the quantum noise in IOB of a pure quantum mechanical system is quite large. By inserting large dephasing processes[15] we can improve the conditions for IOB but not for obtaining squeezing.

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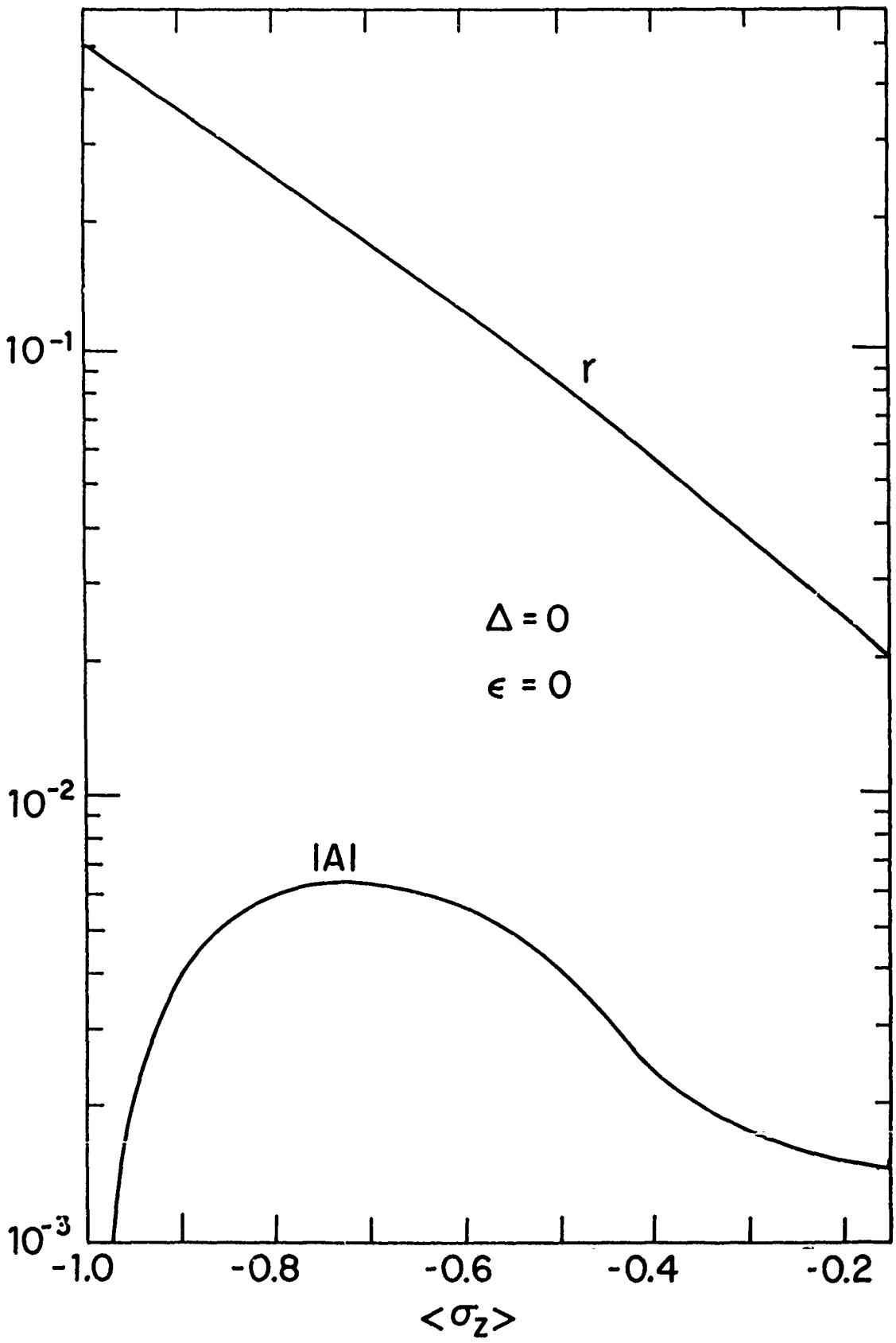
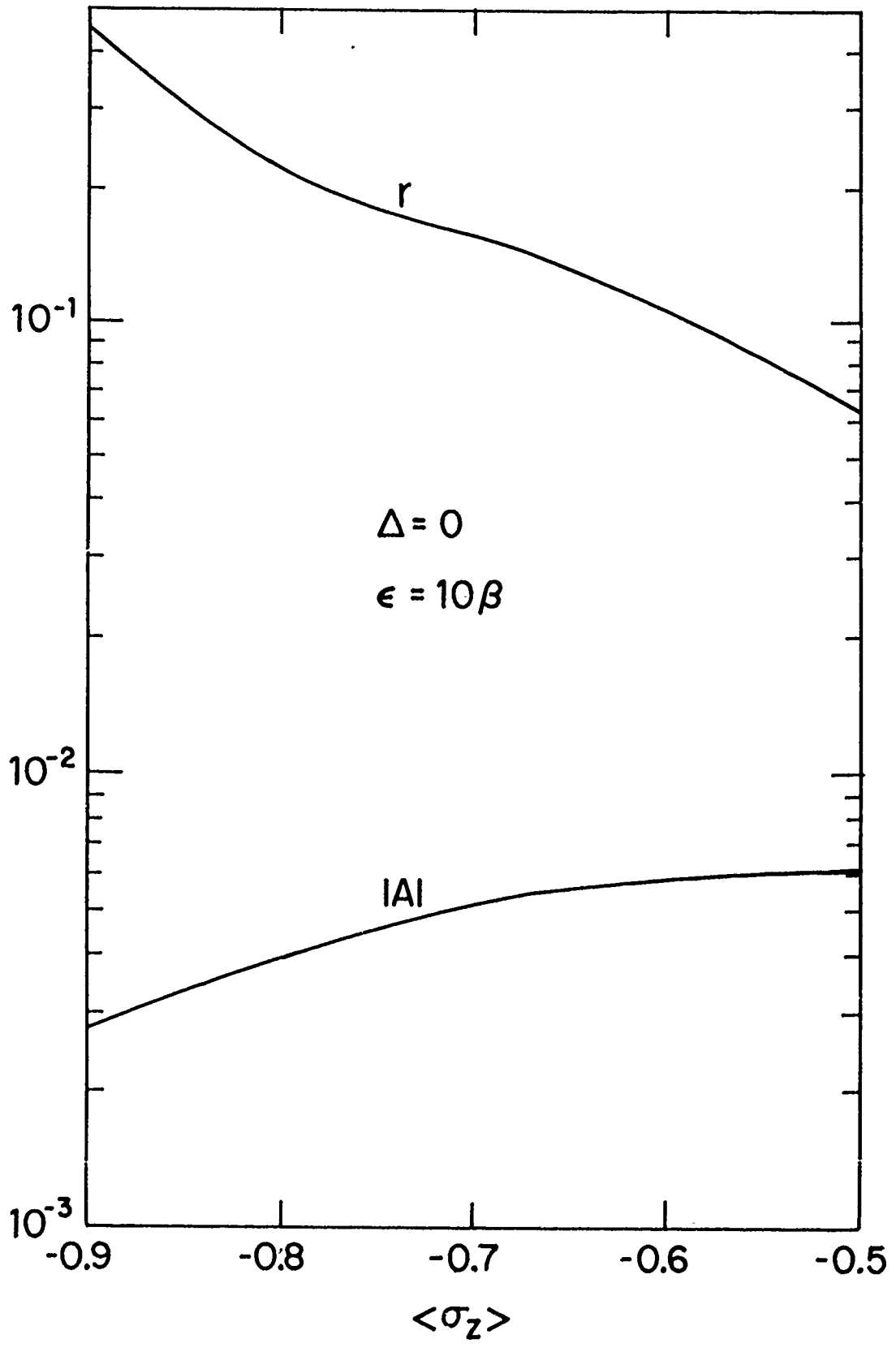
FIGURE CAPTIONS

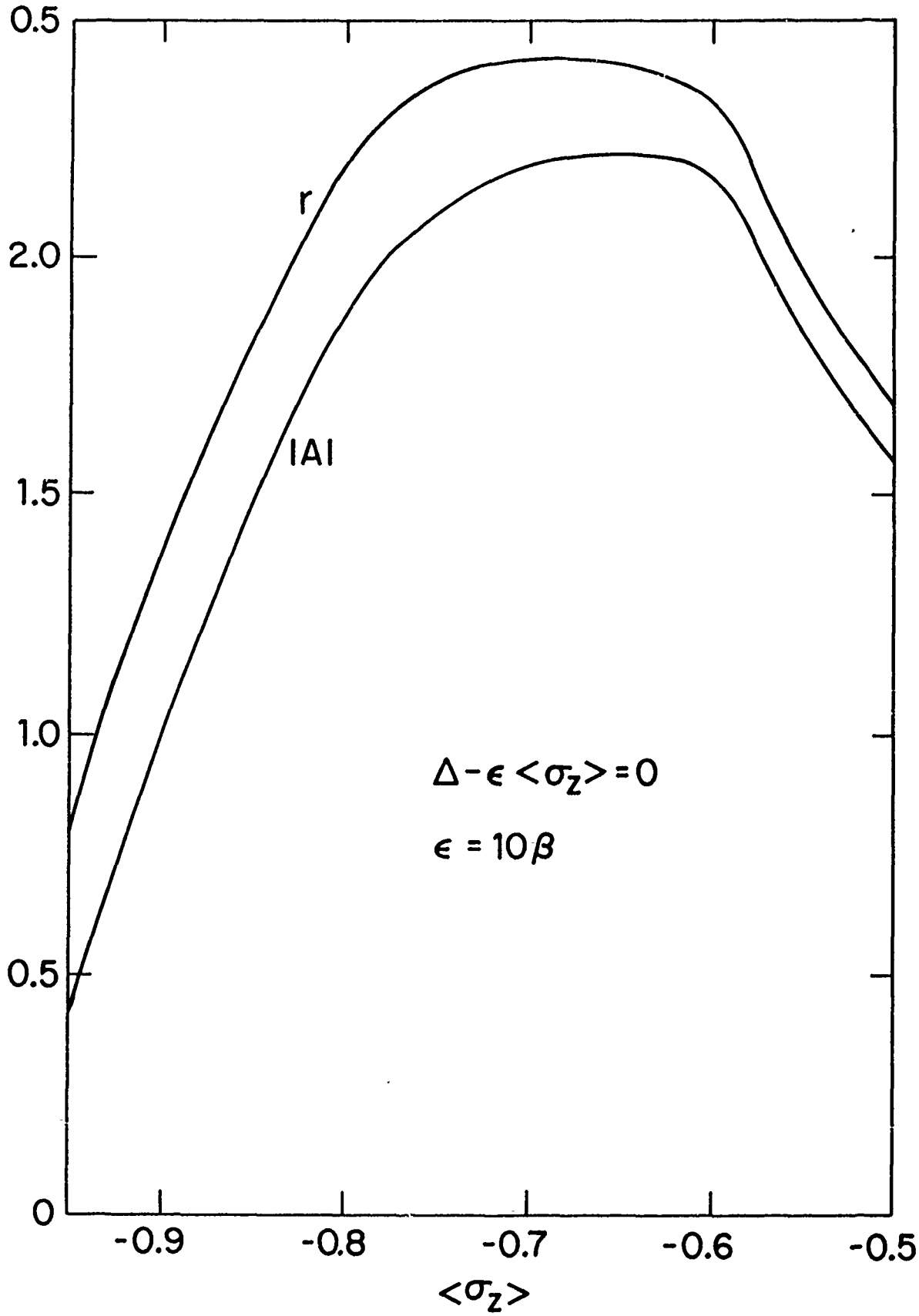
Fig. 1. The parameters r and $|A|$ are described as a function of $< \sigma_z >$ for the ordinary Bloch equations on resonance ($\Delta = 0$). The calcualtions have been made according to Eqs. (36).

Fig. 2. The parameters r and $|A|$ are described as a function of σ_z (for $|\sigma_z| \geq 0.5$) for IOB on resonance ($\Delta = 0$) where $\epsilon = 10\beta$.

Fig. 3. The parameters r and $|A|$ are described as a function of $< \sigma_z >$ (for $|\sigma_z| \geq 0.5$) for IOB in which $\Delta - \epsilon < \sigma_z >= 0$ and $\epsilon = 10\beta$.


10^{-1}
10^{-2}
10^{-3}
r
Δ = 0
ϵ = 0
|A|
-1.0
-0.8
-0.6
-0.4
-0.2
$\langle\sigma_z\rangle$


r
10^{-1}
$\Delta = 0$
$\epsilon = 10\beta$
10^{-2}
|A|
10^{-3}
-0.9
-0.8
-0.7
-0.6
-0.5
$\langle\sigma_z\rangle$


0.5
2.0
1.5
1.0
0.5
0
r
|Δ|
Δ-ε<σz>=0
ε = 10β
-0.9
-0.8
-0.7
-0.6
-0.5
<σz>